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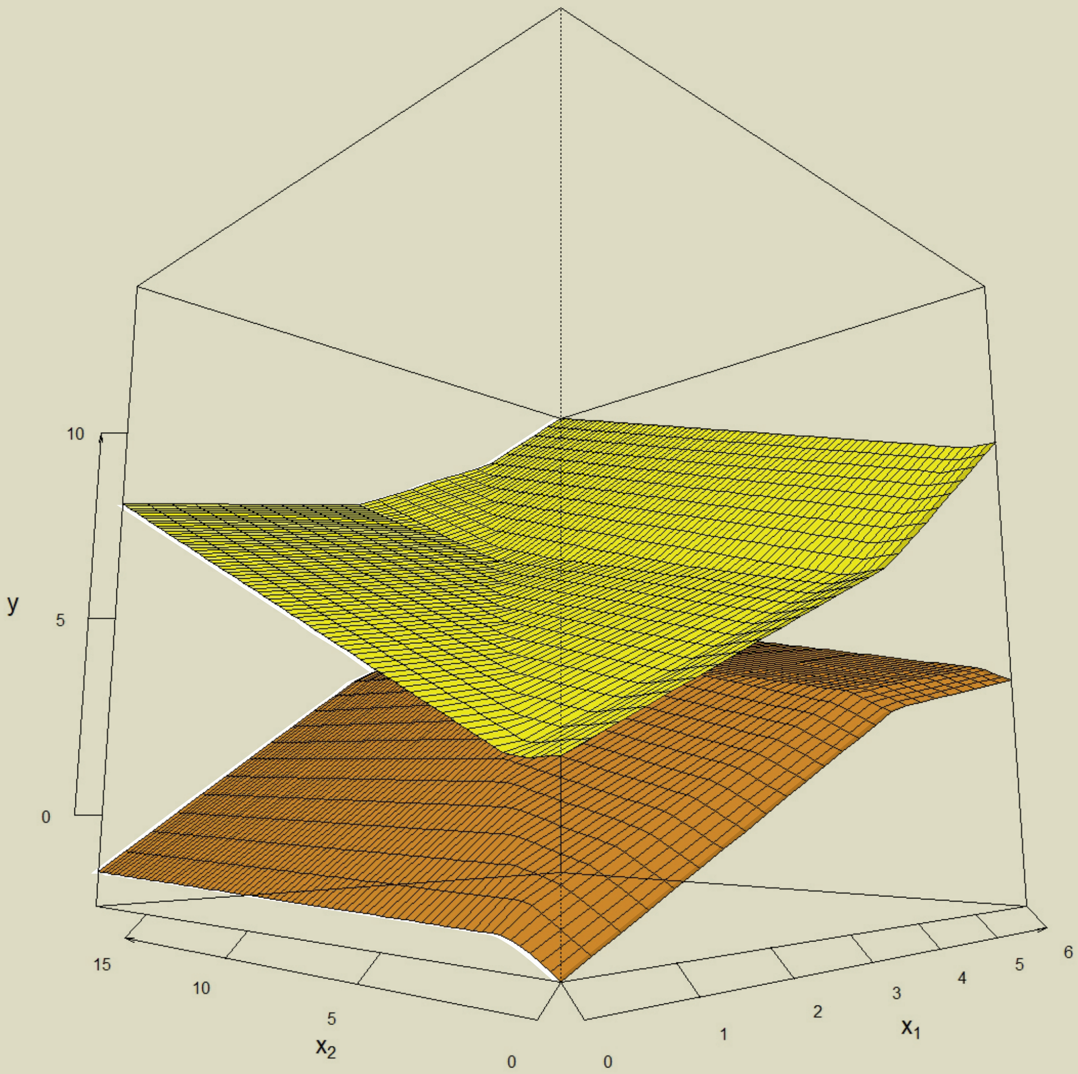
Operations Research

with Statistical Applications

Seksan Kiatsupaibul



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Preface

This book has been developed from lecture notes for a course in operations research at the Department of Statistics, Chulalongkorn Business School. The objective is to provide the audience with basic tools to analyze decision making problems under uncertainty. Since the group of statistics students is a primary target of this book, a number of examples in statistical applications are also included.

The presentation in each chapter usually starts with a simple decision making case study, followed by a sequence of logical arguments that leads to solution methods. Each case study is kept in its simplest form and does not attempt to be over realistic. This style of presentation is designed so that the audience can grasp the problem structure as quickly as possible, then use it to induce an insight into the solution method. More efforts are spent on the solution methods. The case studies also play a key role as tokens that remind the audience of the solution concepts.

The first three chapters, as in other operations research books, are devoted to linear programming formulations. A linear program is very rich in its structure and has always been a good place to start. Most mathematical programming concepts are introduced in this chapter and real world applications are related. The simplex method, a notable practical solution method for a linear program, is postponed to Chapter 4. This is done so that the author can have more space to elaborate and put more emphasis on the solution aspects of operations research. Chapter 5 introduces the concept of duality in linear programming. In Chapter 6, the presentation departs from a traditional operations research textbook by discussing an application of linear programming to statistics. Quantile regression is specifically discussed. In Chapter 7, uncertainty is introduced into decision making problems. In this chapter, the author shows how linear programming, deterministic in its nature, can address the problem with uncertainty. A quadratic program in a form of a Markowitz model is also introduced as an alternative to addressing uncertainty. In Chapter 8, convex programming and its solution techniques are introduced. Examples on how to fit a logistic regression model in statistics by convex programming solution techniques are given. Chapters 9 and 10 are dedicated to sequential decision making problems. Chapter 9 discusses deterministic sequential decision models. Chapter 10 deals with stochastic sequential decision models. Chapters 11 and 12 discuss some stochastic systems. Chapter 11 introduces basic queuing theory and sets the stage for the simulation techniques in Chapter 12. In the appendix, the prerequisites on vector calculus and probability are provided.

The author would like to thank Professor Robert L. Smith for his guidance in operations research over the years at the University of Michigan and up until now. The author would also like to thank Professor Anthony Hayter who provided insightful comments regarding applications of operations research to problems in his research collaboration with the author. Professor Hayter also helped read through the manuscript, for which the author is very grateful. The early draft of this text has been used in the fourth year undergraduate classes at the Department of Statistics, Chulalongkorn University with graduate students as the teaching assistants. In particular, two graduate students, Pariyakorn Maneekul and Chutimon Sindhuprama, have been very helpful in reviewing the text and checking exercises. Special thanks to my brother, Khomsan Kiatsupaibul, who helped design the book cover. The author thanks Chulalongkorn Business School for its remarkable working environment. It was the main factor that made this small project possible. Any shortcomings of this book belong to the author alone.

Seksan Kiatsupaibul
December, 2018
Chulalongkorn Business School
Bangkok, Thailand

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Chapter 1

Introduction to Operations Research

1.1 Introduction

This book is written for an introductory course in operations research. Students will learn about optimization techniques for solving decision making problem. In addition, fitting a statistical model is here categorized as a decision problem that we would also like to solve.

Students should be aware that there exists an interplay between operations research and statistics. Operations research requires probability and statistics for modeling and analyzing uncertainty in its decision problems. In turn, statistics utilizes optimization techniques in operations research to fit its models. In this book, we also investigate this interplay between operations research and statistics.

The topics covered in this book include optimization models, linear programming, convex programming, dynamic programming, stochastic systems and simulation. In general, the presentation starts with a problem or a case study. Then a relevant operations research model is introduced to describe the problem, followed by model analyses and solution techniques. The presentation ends with a conclusion and insights obtained from the solution.

Students are encouraged to use a spreadsheet program such as Excel to implement the models discussed. A hands-on experience building a spreadsheet model to solve a problem will enhance the students' understanding of the subject.

1.2 Operations Research

Operations research is a discipline that emphasizes developing and employing quantitative techniques to solve decision making problems. In the past, such quantitative analysis was restricted to problems with simple structures, due to limited computational resources. However, today information technology opens ways for computationally intensive quantitative techniques to be implemented. Novel computationally intensive tools have been developed to take advantage of these newly available resources. As a result, many complex problems that in the past were considered intractable have been re-considered and solved. For students, today easy-to-use spreadsheet programs, such as Excel, have these quantitative tools built into their system. Solving a complex problem is no longer out of reach.

The ultimate goal of this course is for students to be able to make a practical decision in the face of uncertainty. To achieve this goal, optimization techniques for decision models without uncertainty, the so-called deterministic decision model, are first introduced. The element of uncertainty is subsuquently added through stochastic modeling. In combination, we obatin more sophisticated models and techniques to solve stochastic decision making problems that are more practical and realistic. The diagram below summarizes this process of development in this book.

$$\begin{array}{c}\text{Optimization}\\ \downarrow \\ \text{Probability and Statistical Tools}\\ \downarrow \\ \text{Optimization under uncertainty}\end{array}$$

Optimization is the template of our decision making problems. When we make a decision, we select the one that provides the best result.

1.3 Optimization Model

A decision problem is composed of *choices* and an *objective*. The *objective* refers to a decision criterion or a performance measure given a choice. The choices are bounded by constraints or policies.

Example 1.1 *A company has two projects - project A* Go Big *and project B* Play Safe. *The management has to choose the one that offers the highest total profit.*

Choices	:	*Project A or B*
Objective	:	*Total profit*
Constraints	:	*Between only A and B*

Here we decide between two choices.

Example 1.2 *A production manager decides the number of units of a product to be produced in order to meet demand within the capacity and specification at the lowest total cost.*

Choices	:	*Producing 1,000 units, 1,001 units, ... of a product*
Objective	:	*Total cost*
Constraints	:	*Meet demand, within the limited resources*

In this example, we decide among thousands of choices.

Each choice is associated with one value of the objective. In Example 1.1, suppose Project A offers THB 10 million of profit, while Project B offers THB 7.5 million of profit. The objectives corresponding to Project A and Project B are THB 10 million and THB 7.5 million, respectively. In Example 1.2, suppose producing 1,000 units incurs the cost of THB 10,000, while producing 2,000 units incurs THB 15,000. The objectives corresponding to producing 1,000 units and producing 2,000 units are THB 10,000 and THB 15,000, respectively.

Note that the objective defined for each choice must be a number, enabling us to compare the objectives among choices. To see the necessity of this restriction, consider the following situation. In a production line, suppose we are choosing between two technologies to be implemented in our manufacturing process. We want to produce a good quality product at a low cost. Both quality and cost are measured in grades: 1 (good), 2 (average), 3 (bad). Suppose we have only technology A and technology B to choose from. Technology A's quality grade is 1, the cost grade is 3. Technology B's quality is 3, the cost is 1. If we take both quality and cost as our objective, which technology would we choose? This simple example shows the necessity of having the objective as a scalar. For example, if we adopt the quality as the objective, we can then easily choose A as the technology of choice.

In case we have more than one performance measure to consider, a possible way is to select one as the objective and put the rest into constraints. Another way is to combine all of those measures into one measure with a mathematical transformation, such as an average or a linear combination.

The decision model discussed above can be put into the following template.

$$\begin{aligned} &\text{Optimize the Objective} \\ \text{s.t.}\quad &\text{Choices within Constraints} \end{aligned} \tag{1.1}$$

The notation *s.t.* abbreviates *subject to*. Optimization can be either maximization or minimization. In what follows, the notations max and min stand for *maximize* and *minimize*, respectively.

Example 1.3 *From Example 1.1, we can write*

$$\begin{array}{rl} \max & \textit{Profit of a project} \\ \textit{s.t.} & \textit{Choosing between project A and project B} \end{array}$$

From Example 1.2, we can write

$$\begin{array}{rl} \min & \textit{Cost of producing } x \textit{ units of a product} \\ \textit{s.t.} & \textit{Producing } x \textit{ units such that} \\ & \textit{(i) meet demand} \\ & \textit{(ii) meet capacity} \end{array}$$

To analyze the problem quantitatively, we have to transform all elements in the above template into numbers. Since the objective is already a number, we only need to transform the choices. In a problem setup, since an optimal choice has not yet been discovered, it is a variable(s) called a *decision variable(s)*. A constraint is a mathematical expression that associates the decision variables to satisfy a policy or a limitation. The following case study will elaborate this point.

1.4 A Case Study

Case 1.1 (Puppy Universal) *Puppy Universal, a manufacturer producing pet foods, plans to launch a new product, aiming at a premium market. The plan requires new investment of THB 50,000 on production fundamentals. To produce one unit of the new product, THB 1.0 is required as a production cost. The product can be sold at THB 2.0 per unit. If the plan succeeds, it will enhance the share of the company in the market of premium grade products. The company wants to decide a suitable production quantity.*

Since the management wants to know the most suitable quantity, the choices are obvious: units of the product to be produced. Assume that we have only three choices, producing 10,000 units, 50,000 units and 100,000 units. Which one is the best? To put it into a mathematical notation, let Q represent a possible choice, called decision variable. The variable can take value of 10,000, 50,000 or 100,000. The problem is to find the best Q.

To determine the best choice, we need an objective. As a business unit, a sensible objective is profit.

$$\max \quad \text{Profit} = (2-1)Q - 50{,}000$$

From this problem setup, the best choice is to produce 100,000 units which will give 150,000 profit.

However, the possible choices Q are not necessarily limited by those three numbers. It can be $0, 1, 2, 3$ and so on. In such a case the number of choices are infinite. If we can produce in subunits, Q can be $0.1, 0.2, \ldots$, which makes the number of choices becomes even larger. In general, we may allow any real number to be our possible choice.

One usually needs policies to reflect physical and economic limitations. Let c be the highest number of units the production line can accommodate, and let s be the expected demand in the market. We then have the following limitations.

$$\begin{aligned} Q &\leq c \\ Q &\leq s \end{aligned}$$

Another sensible policy can be that the profit gained from selling the product must cover the fixed cost. Therefore,

$$(2-1)Q \geq 50{,}000$$

or

$$Q \geq 50{,}000$$

These are the constraints that were mentioned earlier.

Combining all the elements together, we can write the problem mathematically as follows:

Program 1.1

$$\max \quad (2-1)Q - 50{,}000$$

$$\begin{aligned} s.t. \quad (2-1)Q &\geq 50{,}000 \\ Q &\leq c \\ Q &\leq s \\ Q &\geq 0 \end{aligned}$$

Note the last constraint $Q \geq 0$. This is called a non-negativity constraint that imposes another physical limitation that we cannot produce a negative number of products.

The optimization model written in this mathematical form is called a *mathematical program*. (The term "program" was used in the military community to mean a schedule to be met.) Some readers might recognize that Program 1.1 has a lot of redundancy, but we do not concern ourselves with this at the moment.

To summarize, Q represents choices, called the decision variable. The objective here is the profit, which is a function of Q. Observe that it is a linear function. The constraints are the limitations imposed on the decision maker. They are expressions written in terms of Q. The constraints we have seen are divided into two parts, *the Left hand side (LHS)* and *the Right Hand Side (RHS)*. LHS relates Q to a quantity representing a measure. RHS is the limitation on that measure. Observe that LHS is also a linear function. This specific mathematical program is called a linear program. Optimization problems do not have to be linear, but problems that are linear are considered to be a