



# AIR CONDITIONING AND VENTILATION

FUNDAMENTAL TO PRACTICAL

ASST. PROF. DR. NOPPARAT KATKHAW



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## **Air Conditioning and Ventilation Fundamental to Practical**

Featuring both SI and I-P units

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# Prefix

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Today's engineering students often struggle with attention during long lectures, dislike note-taking, and miss recording key concepts. These issues are not solely the students' fault; rather, they point out the need for instructors to adapt their teaching strategies. However, many university-level instructors — especially in engineering — may not be well-equipped with educational psychology or classroom engagement techniques.

One effective solution I've found through years of teaching and professional experience is the use of a well-designed textbook. A book with clear explanations, real-world examples, and dual-unit presentation can empower students to learn independently and grasp complex systems with confidence.

While the ASHRAE Handbooks, SMACNA, and CARRIER references are invaluable in professional HVAC practice, they are often too comprehensive and fragmented for structured teaching. Therefore, this book was developed to bridge that gap. Drawing from my two decades of experience in HVAC system design, university teaching, and on-site engineering instruction, this book presents essential HVAC content in a streamlined and practical format.

The book integrates real-world examples, including both SI and I-P units, to align with global usage patterns. Mixed units often appear in engineering documents across the US, Europe, and Asia, so this approach ensures the content is flexible and globally relevant.

Examples' numerical values may slightly deviate from hand-calculated results due to the use of automated software equations that do not round intermediate steps. This decision reflects the accuracy and workflow common in professional engineering practice.

I hope this book supports learners, educators, and professionals alike — making complex HVAC knowledge both accessible and applicable.

**Assistant Professor Dr. Nopparat Katkhaw**  
April 2025



# Acknowledgments

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I humbly dedicate this work to my father, Mr. Dilok, and my mother, Mrs. Chaliao Katkhaw, who gave me both life and the opportunity for education. I also pay my deepest respects to my teachers—Mr. Samart Methnawin, Mr. Kiri Thirasin, and Mr. Atipong Nantaphan—and all my former supervisors who imparted to me their expertise in the field of air-conditioning.

I would also like to thank my colleagues, my family, my wife, and my child for their constant encouragement and unwavering support, which made this work possible.

**Assistant Professor Dr. Nopparat Katkhaw**  
April 2025



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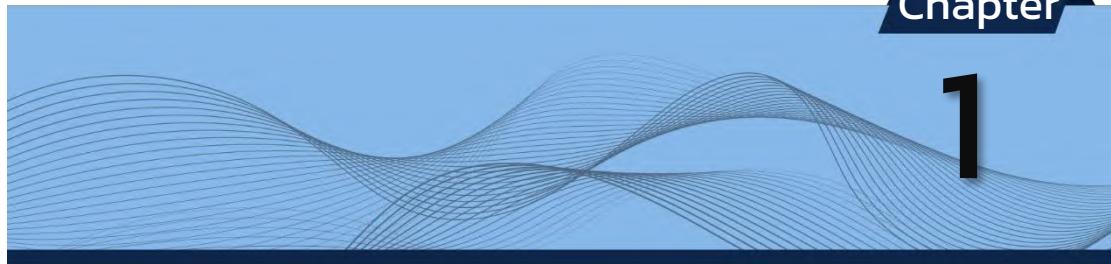
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## **Index**

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Psychrometrics involves the analysis of air thermodynamic properties for application in heating, ventilation, air-conditioning, and refrigeration (HVAC&R) systems. This chapter explains the properties of moist air and the basic processes related to it in HVAC applications, including ideas about thermal comfort and environmental factors that influence how comfortable people feel.

## 1.1 Dry Air and Moist Air

---

At standard atmospheric conditions, ambient air contains water vapor as part of its composition. This air is defined as a mixture of dry air and water vapor, commonly referred to as moist air. Moist air can contain water vapor up to its saturation limit, referred to as saturated air. If additional water droplets are introduced into air that is already saturated, the droplets will not evaporate but will either settle or remain suspended as mist. When the temperature of the saturated air decreases, condensation occurs, forming fog or dew.

Dry air and water vapor behave independently. Under typical atmospheric conditions, dry air remains in the gaseous state and cannot be liquefied by pressure alone. Therefore, the ideal gas law is applicable for dry air analysis. In contrast, water vapor exists in gaseous, liquid, or solid phases and can condense under pressure alone. As a result, its thermodynamic properties are more complex and must be determined experimentally.

Dry air is defined as air free of water vapor and contaminants. The thermodynamic properties of dry air and water vapor vary with altitude, temperature, and pressure. At sea level, dry air usually contains these gases in the following amounts: nitrogen (78.084%), oxygen (20.947%), argon (0.934%), carbon dioxide (0.0314%), neon (18.2 parts per million), helium (5.2 parts per million), methane (2.0 parts per million), krypton (1.1 parts per million), hydrogen (0.05 parts per million), xenon (0.009 parts per million), ozone (0.007 parts per million), nitrogen dioxide (0.002 parts per million), and iodine (0.001 parts per million), plus small amounts of carbon monoxide and ammonia (Harrison, 1965).

## 1.2 Standard Atmosphere

Air temperature and barometric pressure vary with elevation. A standard atmosphere is established as a reference baseline for calculations at various elevations above sea level. Standard atmospheric conditions at sea level are defined as 15°C (59°F) and 101.325 kPa (14.696 psia). Temperature is assumed to decrease linearly with elevation. Based on this standard atmosphere, the relationship between pressure, temperature, and elevation can be expressed using standard atmosphere equations (ASHRAE Handbook 2021).

$$p = 101.325(1 - 2.25577 \times 10^{-5}Z)^{5.2559} \quad \text{SI} \quad (1)$$

$$p = 14.696(1 - 6.8754 \times 10^{-6}Z)^{5.2559} \quad \text{IP} \quad (1)$$

$$t = 15 - 0.0065Z \quad \text{SI} \quad (2)$$

$$t = 59 - 0.0035662Z \quad \text{IP} \quad (2)$$

where  $p$  = barometric pressure, kPa (psia)

$Z$  = altitude, m (ft)

$t$  = temperature, °C (°F)

Equations (1) and (2) are valid only for elevations from –5000 m (–15,000 ft) to 11,000 m (33,000 ft) above sea level. The pressure-elevation relationship described by these equations is used to support psychrometric property calculations.

At sea level, the standard atmosphere assumes dry air with a composition that remains constant with elevation. However, actual atmospheric conditions may vary with geographic location, seasonal changes, and weather phenomena. These equations provide a reliable basis for calculating thermodynamic properties of air and moist air mixtures. They are especially useful in psychrometric analysis and in the design of HVAC systems for varying elevations, where changes in pressure and temperature significantly affect equipment performance and capacity sizing.

## 1.3 Water Vapor Saturation Pressure

Determining the properties of moist air requires obtaining the saturation pressure of water vapor, using air temperature to retrieve saturation pressure from steam tables. Alternatively, the following equation may be used (Hyland and Wexler, 1983).

**For the temperature range of  $-100^{\circ}\text{C}$  to  $0^{\circ}\text{C}$  ( $-148^{\circ}\text{F}$  to  $32^{\circ}\text{F}$ );**

$$\ln p_{ws} = C_1/T + C_2 + C_3T + C_4 T^2 + C_5T^3 + C_6T^4 + C_7\ln T \quad \text{SI and IP} \quad (3)$$

where SI unit

$$\begin{aligned} p_{ws} &= \text{saturation pressure, Pa} \\ T &= \text{absolute temperature,} \\ &\quad \text{K} = ^{\circ}\text{C} + 273.15 \\ C_1 &= -5.6745359 \times 10^3 \\ C_2 &= 6.3925247 \\ C_3 &= -9.677843 \times 10^{-3} \\ C_4 &= 6.2215701 \times 10^{-7} \\ C_5 &= 2.0747825 \times 10^{-9} \\ C_6 &= -9.484024 \times 10^{-13} \\ C_7 &= 4.1635019 \end{aligned}$$

IP unit

$$\begin{aligned} p_{ws} &= \text{saturation pressure, psia} \\ T &= \text{absolute temperature,} \\ &\quad ^{\circ}\text{R} = ^{\circ}\text{F} + 459.67 \\ C_1 &= -1.0214165 \times 10^4 \\ C_2 &= -4.8932428 \\ C_3 &= -5.3765794 \times 10^{-3} \\ C_4 &= 1.9202377 \times 10^{-7} \\ C_5 &= 3.5575832 \times 10^{-10} \\ C_6 &= -9.0344688 \times 10^{-14} \\ C_7 &= 4.1635019 \end{aligned}$$

**For the temperature range of  $0^{\circ}$  to  $200^{\circ}\text{C}$  ( $32^{\circ}\text{F}$  to  $392^{\circ}\text{F}$ );**

$$\ln p_{ws} = C_8/T + C_9 + C_{10}T + C_{11} T^2 + C_{12}T^3 + C_{13}\ln T \quad \text{SI and IP} \quad (4)$$

where SI unit

$$\begin{aligned} C_8 &= -5.8002206 \times 10^3 \\ C_9 &= 1.3914993 \\ C_{10} &= -4.8640239 \times 10^{-2} \\ C_{11} &= 4.1764768 \times 10^{-5} \\ C_{12} &= -1.4452093 \times 10^{-8} \\ C_{13} &= 6.5459673 \end{aligned}$$

IP unit

$$\begin{aligned} C_8 &= -1.0440397 \times 10^4 \\ C_9 &= -1.1294650 \times 10^1 \\ C_{10} &= -2.7022355 \times 10^{-2} \\ C_{11} &= 1.2890360 \times 10^{-5} \\ C_{12} &= -2.478068 \times 10^{-9} \\ C_{13} &= 6.5459673 \end{aligned}$$

The saturation pressure of water vapor ( $p_{ws}$ ) refers to the saturation pressure of pure water vapor without any mixture of air. This pressure depends solely on temperature and differs slightly from the actual vapor pressure ( $p_s$ ) of saturated moist air at the same temperature. Therefore, if the actual vapor pressure of saturated air is known, other thermodynamic properties of the water vapor can be determined.

## 1.4 Ideal Gas

In the analysis of moist air, the ideal gas equation can accurately determine the components of dry air, water vapor, and moist air, especially at pressures near atmospheric pressure. For a mixture of ideal gases, Dalton's Law states that all gas and vapor molecules occupy the same volume, and the total pressure of the mixture is equal to the sum of the partial pressures of each component. Therefore, the pressure of moist air is equal to the sum of the partial pressures of dry air and water vapor.

When air and water vapor behave as ideal gases, the ideal gas law and Dalton's law describe the behavior of the mixture as follows:

Dry air ( $da$ );	$p_{da}V = n_{da}R_{da}T$	SI and IP	(5)
-------------------	---------------------------	-----------	-----

Water vapor ( $w$ );	$p_wV = n_wR_wT$	SI and IP	(6)
----------------------	------------------	-----------	-----

where

$p$  = partial pressure, kPa (psia)

$V$  = total mixture volume,  $\text{m}^3$  ( $\text{ft}^3$ )

$n$  = number of moles

$R$  = universal gas constant,  $8314.472 \text{ J/kmol}\cdot\text{K}$ , ( $1545.349 \text{ ft}\cdot\text{lbf/lb}_{\text{mol}}\cdot{}^{\circ}\text{R}$ )

$T$  = absolute temperature, K ( ${}^{\circ}\text{R}$ )

## 1.5 Properties of Moist Air

Using the ideal gas law and the saturation pressure equation for water vapor, the ASHRAE Handbook (2021) provides the following methods to determine other properties of moist air:

### 1.5.1 Total Barometric Pressure, $p$

Typical measurements of moist air pressure indicate that the measured value is the total barometric pressure, which equals the sum of the partial pressures of dry air and water vapor, as expressed in the following equation:

$p = p_{da} + p_w$	SI and IP	(7)
--------------------	-----------	-----

Substituting these values into the ideal gas law yields the properties of moist air as follows:

$$(p_{da} + p_w)V = (n_{da} + n_w)RT \quad \text{SI and IP} \quad (8)$$

where  $n_{da} + n_w$  is the total number of moles in the moist air.

Based on Equations (5) through (8), the mole fractions ( $x$ ) of dry air and water vapor in moist air are determined as follows:

$$x_{da} = p_{da}/(p_{da} + p_w) = p_{da}/p \quad \text{SI and IP} \quad (9)$$

$$x_w = p_w/(p_{da} + p_w) = p_w/p \quad \text{SI and IP} \quad (10)$$

### 1.5.2 Humidity Ratio or Mixing Ratio, $W$

The humidity ratio is defined as the ratio of the mass of water vapor ( $M_w$ ) to the mass of dry air ( $M_{da}$ ), as given by the following equation:

$$W = M_w/M_{da} \quad \text{SI and IP} \quad (11)$$

The humidity ratio ( $W$ ) is expressed in  $\text{kg}_w/\text{kg}_{da}$  ( $\text{lb}_w/\text{lb}_{da}$  in IP units) and is equal to the ratio of mole fractions ( $x_w/x_{da}$ ) times the ratio of molecular weights (18.015268/28.966), as shown in the following equation:

$$W = 0.621945 x_w/x_{da} \quad \text{SI and IP} \quad (12)$$

Based on Equations (9), (10), and (12), the humidity ratio is determined as follows:

$$W = 0.621945 p_w/p_{da} = 0.621945 \frac{p_w}{p - p_w} \quad \text{SI and IP} \quad (13)$$

### 1.5.3 Saturation Humidity Ratio, $W_s$

The saturated humidity ratio is the humidity ratio of air saturated with respect to water or ice at the same temperature and pressure. It is determined by the following equation:

$$W_s = 0.621945 \frac{p_{ws}}{p - p_{ws}} \quad \text{SI and IP} \quad (14)$$

### 1.5.4 Specific Humidity, $\gamma$

Specific humidity is the ratio of water vapor mass to moist air mass, as given by the following equation:

$$\gamma = M_w/(M_w + M_{da}) \quad \text{SI and IP} \quad (15)$$

The following equation also expresses it in terms of the humidity ratio:

$$\gamma = W/(1 + W) \quad \text{SI and IP} \quad (16)$$

### 1.5.5 Absolute Humidity, $d_v$

Absolute humidity, or water vapor density, is the ratio of water vapor mass to moist air volume, as given by the following equation:

$$d_v = M_w/V \quad \text{SI and IP} \quad (17)$$

### 1.5.6 Density, $\rho$

Density is the mass-to-volume ratio of moist air, as shown in the following equation:

$$\rho = (M_w + M_{da})/V = (1/v)(1 + W) \quad \text{SI and IP} \quad (18)$$

where  $v$  is the specific volume of moist air, in  $\text{m}^3/\text{kg}_{da}$  ( $\text{ft}^3/\text{lb}_{da}$  in IP units)

### 1.5.7 Specific Volume, $v$

The specific volume is the volume of moist air per unit mass of dry air, as shown in the following equation:

$$v = V/M_{da} = V/(28.966n_{da}) \quad \text{SI and IP} \quad (19)$$

From the ideal gas equation (Equation 5) and the total pressure equation (Equation 7), the following equation is derived:

$$v = \frac{RT}{28.966(p - p_w)} = \frac{R_{da}T}{p - p_w} \quad \text{SI and IP} \quad (20)$$

From Equation (9), the following is obtained:

$$v = \frac{RT(1 + 1.607858W)}{28.966p} = \frac{R_{da}T(1 + 1.607858W)}{p - p_w} \quad \text{SI and IP} \quad (21)$$

Substituting the value of  $R$  into the equation, the following equation is obtained:

$$v = 0.287042(t + 273.15)(1 + 1.607858W)/p \quad \text{SI} \quad (22)$$

$$v = 0.370486(t + 459.67)(1 + 1.607858W)/p \quad \text{IP} \quad (22)$$

where  $v$  = specific volume,  $\text{m}^3/\text{kg}_{da}$  ( $\text{ft}^3/\text{lb}_{da}$ )  
 $t$  = dry bulb temperature,  $^{\circ}\text{C}$  ( $^{\circ}\text{F}$ )  
 $W$  = humidity ratio,  $\text{kg}_w/\text{kg}_{da}$  ( $\text{lb}_w/\text{lb}_{da}$ )  
 $p$  = total pressure,  $\text{kPa}$  ( $\text{psia}$ )

### 1.5.8 Degree of Saturation, $\mu$

The degree of saturation is the ratio of the actual humidity ratio ( $W$ ) to the saturated humidity ratio ( $W_s$ ) at the same temperature and pressure, as shown in the following equation:

$$\mu = W/W_s \quad \text{SI and IP} \quad (23)$$

For dry air, the degree of saturation and relative humidity are zero.

### 1.5.9 Relative Humidity, $\phi$

Relative humidity is the ratio of the mole fraction of water vapor to the mole fraction of saturated water vapor and equals the ratio of the actual water vapor partial pressure to the saturation vapor pressure at the same dry-bulb temperature and pressure, as shown in the following equation:

$$\phi = x_w/x_{ws} = p_w/p_{ws} \quad \text{SI and IP} \quad (24)$$

Alternatively, it is expressed in terms of the degree of saturation as follows:

$$\phi = \frac{\mu}{1 - (1 - \mu)(p_{ws}/p)} \quad \text{SI and IP} \quad (25)$$

### 1.5.10 Dew-Point Temperature, $t_d$

Dew point temperature is the temperature at which air becomes saturated at the prevailing pressure. At this temperature, water vapor begins to condense. A common example is the formation of water droplets on the surface of a glass of ice water. This phenomenon occurs because the glass surface is at a low temperature, causing the moist air to cool to its dew point, leading to condensation.

The saturation vapor pressure at the dew point temperature ( $p_{ws}(t_d)$ ), is determined using the following equation:

$$p_{ws}(t_d) = p_w = (p/W)(0.621945 + W) \quad \text{SI and IP} \quad (26)$$

where  $p_w$  = partial pressure of water vapor

$p_{ws}(t_d)$  = saturation vapor pressure at the dew point temperature

The saturation vapor pressure at the dew point temperature ( $p_{ws}(t_d)$ ), is obtained from Equations (3) and (4). Peppers' equation (1988) determines the dew point temperature as follows:

**Between dew points of 0°C to 93°C (32°F to 200°F):**

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}p_w^{0.1984} \quad \text{SI and IP} \quad (27)$$

**Below 0°C:**

$$t_d = 6.09 + 12.608\alpha + 0.4959\alpha^2 \quad \text{SI} \quad (28)$$

$$t_d = 90.12 + 26.142\alpha + 0.8927\alpha^2 \quad \text{IP} \quad (28)$$

where SI units

	SI units	IP units
$t_d$	= dew-point temperature, °C	$t_d$ = dew-point temperature, °F
$\alpha$	= $\ln p_w$	$\alpha$ = $\ln p_w$
$p_w$	= partial pressure, kPa	$p_w$ = partial pressure, psia
$C_{14}$	= 6.54	$C_{14}$ = 100.45
$C_{15}$	= 14.526	$C_{15}$ = 33.193
$C_{16}$	= 0.7389	$C_{16}$ = 2.319
$C_{17}$	= 0.09486	$C_{17}$ = 0.17074
$C_{18}$	= 0.4569	$C_{18}$ = 1.2063

### 1.5.11 Enthalpy of Moist Air, $h$

The enthalpy of moist air is the sum of dry-air enthalpy and water vapor enthalpy, as shown in the following equation:

$$h = h_{da} + Wh_g \quad \text{SI and IP} \quad (29)$$

where  $h_{da}$  = specific enthalpy for dry air, kJ/kg<sub>da</sub> (Btu/lb<sub>da</sub>)

$h_g$  = specific enthalpy for saturated water vapor, kJ/kg<sub>da</sub> (Btu/lb<sub>da</sub>)

The ASHRAE Handbook (2021) gives the equation to estimate the enthalpy of dry air at different temperatures:

$$h_{da} \approx 1.006t \quad \text{SI} \quad (30)$$

$$h_{da} \approx 0.240t \quad \text{IP} \quad (30)$$

For water vapor at 0°C (32°F), it has an enthalpy of 2501 kJ/kg (1075 Btu/lb<sub>w</sub>) and a specific heat capacity of 1.86 kJ/kg·K (0.444 Btu/lb<sub>w</sub>·F). Therefore, the enthalpy of water vapor is approximated using the following equation:

$$h_g \approx 2501 + 1.86t \quad \text{SI} \quad (31)$$

$$h_g \approx 1075 + 0.444(t - 32) \quad \text{IP}$$

$$h_g \approx 1061 + 0.444t \quad \text{IP} \quad (31)$$

where  $t$  = dry-bulb temperature, °C (°F)

Substituting  $h_{da}$  and  $h_g$  into the equation, the specific enthalpy of moist air is obtained as follows:

$$h = 1.006t + W(2501 + 1.86t) \quad \text{SI} \quad (32)$$

$$h = 0.240t + W(1061 + 0.444t) \quad \text{IP} \quad (32)$$

### 1.5.12 Thermodynamic Wet-Bulb Temperature, $t^*$

The wet-bulb temperature process can be illustrated by considering moist air flowing into a long, insulated duct that contains water. As the air moves through the duct, water evaporates into the air, increasing both the humidity ratio and the enthalpy of the air until the air becomes saturated at the water temperature, without heat transfer. This process is known as adiabatic saturation. The saturated air exits the duct at a temperature *called the thermodynamic wet-bulb temperature ( $t^*$ )*.

From the first law of thermodynamics, the following is obtained:

$$h + (W_s^* - W)h_w^* = h_s^* \quad \text{SI and IP} \quad (33)$$

where  $W_s^*$ ,  $h_w^*$  and  $h_s^*$  are the properties at temperature  $t^*$  and  $h_w^*$  is the enthalpy of the water that evaporates into the air.

When using a psychrometer to measure wet-bulb temperature, air passes over the wetted tip of the bulb. Water evaporates into the air, drawing heat from the water and bulb, lowering its temperature. The procedure is not an adiabatic saturation process because heat is exchanged with the environment. Therefore, the measured temperature is called the psychrometric wet-bulb temperature. In the case of a sling psychrometer, the instrument must be rotated to maintain airflow over the bulb at approximately 0.45 m/s (90 fpm) and long enough to reach thermal equilibrium.

Moist air at near-atmospheric pressure has a thermodynamic wet-bulb temperature approximately equal to the psychrometric wet-bulb temperature. Therefore, Equation (33) is still useful for determining other air properties.

Water enthalpy increases with temperature. At 0°C (32°F), it has zero enthalpy and a specific heat capacity of approximately 4.186 kJ/kg·K (1.0 Btu/lb<sub>w</sub>·°F). Therefore, water enthalpy at different temperatures is approximated as follows:

$$h_w^* \approx 4.186t^* \quad \text{SI} \quad (34)$$

$$h_w^* \approx t^* - 32 \quad \text{IP} \quad (34)$$

From Equations (33) and (34), the humidity ratio is determined as follows:

$$W = \frac{(2501 - 2.326t^*)W_s^* - 1.006(t - t^*)}{2501 + 1.86t - 4.18t^*} \quad \text{SI} \quad (35)$$

$$W = \frac{(1093 - 0.556t^*)W_s^* - 0.240(t - t^*)}{1093 + 0.444t - t^*} \quad \text{IP} \quad (35)$$

where  $t$  = air temperature, °C (°F)

$t^*$  = wet-bulb temperature, °C (°F)

Equation (35) is used to determine the wet-bulb temperature. However, it is nonlinear and requires iterative solving. Roland Stull (2011) proposed an equation for calculating the wet-bulb temperature based on the dry-bulb temperature and relative humidity, as shown in Equation (36). This equation is applicable for moist air at a pressure of 101.325 kPa. It provides accurate results when the relative humidity is between 5%RH and 99%RH and the temperature ranges from -20°C to 50°C. The associated error ranges from -1°C to +0.65°C, with a mean absolute of error less than 0.3°C.

$$t^* = t \operatorname{atan} [0.151977 (\text{RH}\% + 8.313659)^{1/2}] + \operatorname{atan}(t + \text{RH}\%) - \operatorname{atan}(\text{RH}\% - 1.676331) + 0.00391838 (\text{RH}\%)^{3/2} \operatorname{atan}(0.023101 \text{RH}\%) - 4.686035 \quad \text{SI} \quad (36)$$

where  $t^*$  = wet-bulb temperature, °C

$t$  = dry-bulb temperature, °C

RH% = relative humidity (100ϕ)

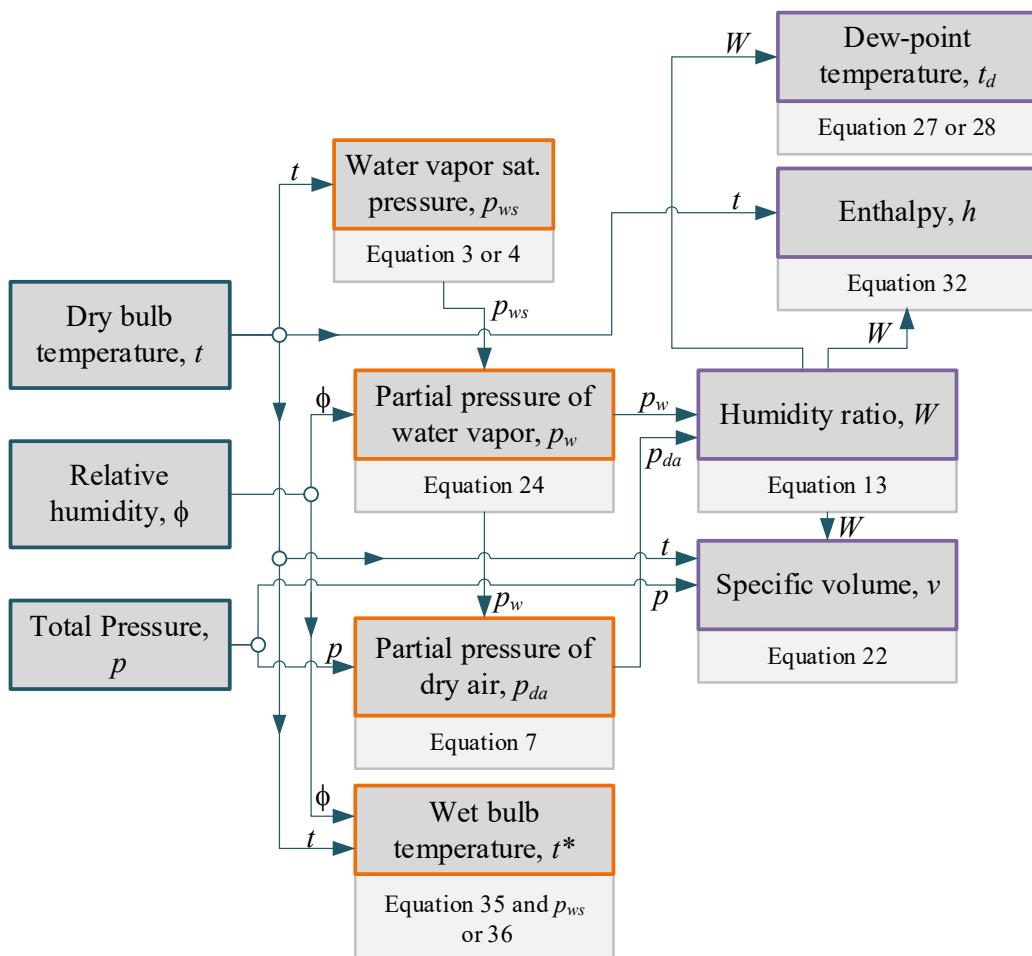
atan = arctangent function expressed in radians

The wet-bulb temperature is limited to a maximum equal to the dry-bulb temperature when the air is saturated.

### 1.5.13 Guidelines for Calculating Moist Air Properties

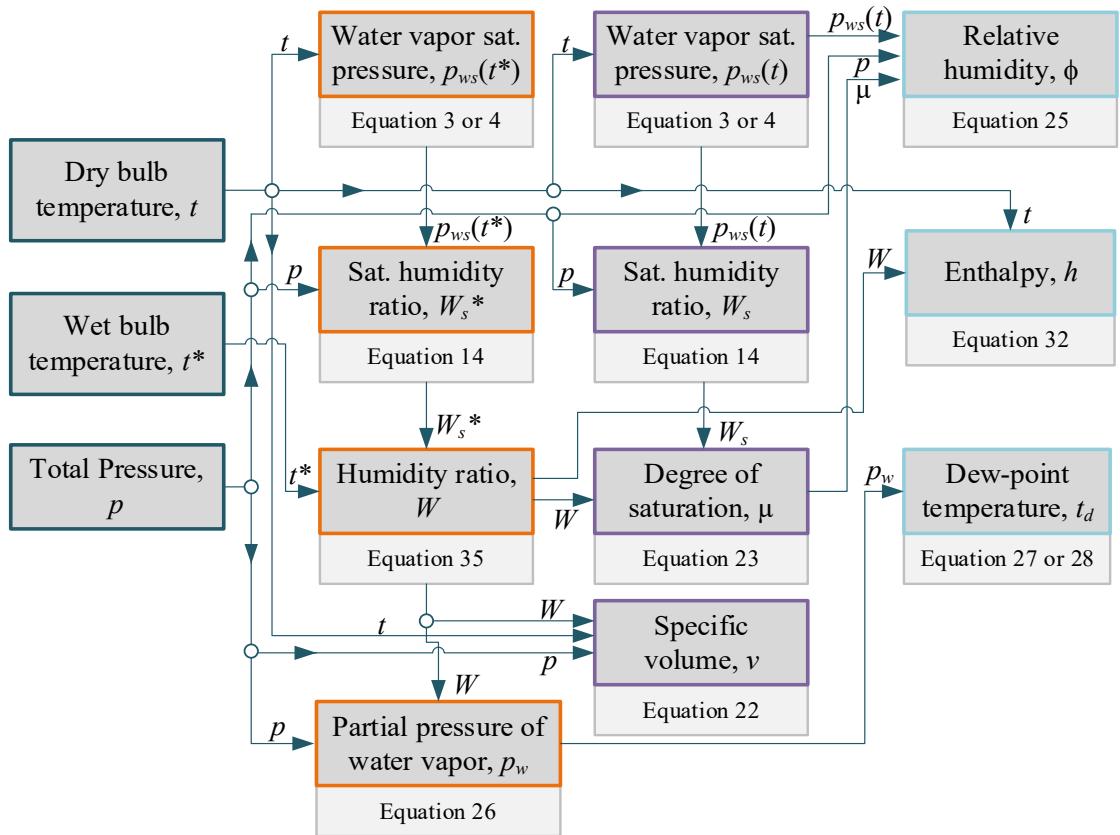
Thermodynamic properties of moist air are interrelated. If two or more properties are known, the remaining properties can be determined. Based on the above equations, two principal methods are used to evaluate moist air properties when a subset is known:

**Method 1:** If the dry-bulb temperature ( $t$ ), relative humidity ( $\phi$ ), and air pressure ( $p$ ) are known remaining properties are determined using the chart in [Figure 1.1](#).



**Fig. 1.1** Flowchart for Determining Moist Air Properties Given  $t$ ,  $\phi$  and  $p$

**Method 2:** If the dry-bulb temperature ( $t$ ), wet-bulb temperature ( $t^*$ ), and air pressure ( $p$ ) are known, remaining properties are determined using the chart in [Figure 1.2](#).



**Fig. 1.2** Flowchart for Determining Moist Air Properties Given  $t$ ,  $t^*$  and  $p$

The equations from both charts can be easily implemented in a spreadsheet such as Microsoft Excel. These equations are applicable to basic air-conditioning calculations. As a preliminary tool, an Excel-based calculation file has been developed, which is available for download at the following link: <https://bit.ly/3rjOuCT>

**Example 1.1****SI Units**

Dry-bulb temperature of air at 101.325 kPa is measured as 25°C (298.15 K), with relative humidity measured at 50%RH ( $\phi = 0.5$ ). Determine the thermodynamic properties of the corresponding moist air.

Given:  $t = 25^\circ\text{C}$  (298.15 K),  $\phi = 0.5$  and  $p = 101.325 \text{ kPa}$ .

Required: The remaining thermodynamic properties of the moist air are calculated.

Solution: The remaining moist air properties are determined using the procedure in the flowchart shown in [Figure 1.1](#), as follows:

- **Saturation vapor pressure ( $p_{ws}$ );** from Equation (4), with  $T = 298.15 \text{ K}$

$$\ln p_{ws} = C_8/298.15 + C_9 + C_{10}298.15 + C_{11}298.15^2 + C_{12}298.15^3 + C_{13}\ln 298.15$$

Using constants  $C$ , we obtain

$$\ln p_{ws} = 8.06124$$

$$\text{and } p_{ws} = e^{\ln p_{ws}} = e^{8.06124} = 3169 \text{ Pa}$$

- **Vapor pressure of moist air ( $p_w$ );** from Equation (24), with  $\phi = 0.5$ :

$$p_w = 0.5p_{ws} = 0.5 \times 3169 = 1584.6 \text{ Pa}$$

- **Partial pressure of dry air ( $p_{da}$ );** from Equation (7), with  $p = 101.325 \text{ kPa}$  and  $p_w = 1.5846 \text{ kPa}$ :

$$p_{da} = p - p_w = 101.325 - 1.5846 = 99.74 \text{ kPa}$$

- **Humidity ratio ( $W$ );** from Equation (13), with  $p = 101.325 \text{ kPa}$  and  $p_w = 1.5846 \text{ kPa}$ :

$$W = 0.621945 \frac{p_w}{p - p_w} = 0.621945 \frac{1.5846}{101.325 - 1.5846}$$

$$W = 0.01 \text{ kg}_w/\text{kg}_{da}$$

- **Specific volume ( $v$ );** from Equation (22), with  $t = 25^\circ\text{C}$ ,  $W = 0.01 \text{ kg}_w/\text{kg}_{da}$  and  $p = 101.325 \text{ kPa}$ :

$$v = 0.287042(t + 273.15)(1 + 1.607858W)/p$$

$$v = 0.287042(25 + 273.15)(1 + 1.607858 \times 0.01)/101.325$$

$$v = 0.8582 \text{ m}^3/\text{kg}_{da}$$

- **Dew point temperature ( $t_d$ )**; from Equation (27):

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18} p_w^{0.1984}$$

From  $p_w = 1.5846$  kPa, we find

$$\alpha = \ln p_w = \ln (1.5846) = 0.4603$$

Substitute the constants  $C$ ,  $p_w$ , and  $\alpha$ :

$$t_d = C_{14} + C_{15}0.4603 + C_{16}0.4603^2 + C_{17}0.4603^3 + C_{18}1.5846^{0.1984}$$

$$t_d = 13.89^\circ\text{C}$$

- **Enthalpy of moist air ( $h$ )**; from Equation (32):

$$h = 1.006t + W(2501 + 1.86t)$$

with  $t = 25^\circ\text{C}$  and  $W = 0.01 \text{ kg}_w/\text{kg}_{da}$ :

$$h = 1.006 \times 25 + 0.01(2501 + 1.86 \times 25)$$

$$h = 50.614 \text{ kJ/kg}_{da}$$

- **Wet-bulb temperature ( $t^*$ )** is obtained from Equation (36) by substituting the dry-bulb temperature and relative humidity:

$$t^* = t \operatorname{atan} [0.151977 (\text{RH}\% + 8.313659)^{1/2}] + \operatorname{atan}(t + \text{RH}\%) - \operatorname{atan}(\text{RH}\% - 1.676331) + 0.00391838(\text{RH}\%)^{3/2} \operatorname{atan}(0.023101\text{RH}\%) - 4.686035$$

$$t^* = 25 \operatorname{atan} [0.151977 (50 + 8.313659)^{1/2}] + \operatorname{atan}(25 + 50) - \operatorname{atan}(50 - 1.676331) + 0.00391838(50)^{3/2} \operatorname{atan}(0.023101 \times 50) - 4.686035$$

$$t^* = 18.00^\circ\text{C}$$

From the ASHRAE psychrometric chart at the same condition, the wet-bulb temperature is  $17.88^\circ\text{C}$ , with a difference of 0.67%.

**Example 1.1****IP Units**

Dry-bulb temperature of air at 14.696 psia is measured as 77°F (536.67°R), with relative humidity measured at 50% ( $\phi = 0.5$ ). Determine the thermodynamic properties of the corresponding moist air.

Given:  $t = 77\text{ }^{\circ}\text{F}$  (536.67°R),  $\phi = 0.5$  and  $p = 14.696\text{ psia}$ .

Required: The remaining thermodynamic properties of the moist air are calculated.

Solution: The remaining moist air properties are determined using the procedure in the flowchart shown in [Figure 1.1](#), as follows:

- **Saturation vapor pressure ( $p_{ws}$ );** from Equation (4), with  $T = 536.67\text{ }^{\circ}\text{R}$

$$\ln p_{ws} = C_8/536.67 + C_9 + C_{10}536.67 + C_{11}536.67^2 + C_{12}536.67^3 + C_{13}\ln 536.67$$

Substituting the constants  $C$ , we obtain

$$\ln p_{ws} = -0.7773$$

$$\text{and } p_{ws} = e^{\ln p_{ws}} = e^{8.0725} = 0.4597$$

- **Vapor pressure of moist air ( $p_w$ );** from Equation (24), with  $\phi = 0.5$ :

$$p_w = 0.5p_{ws} = 0.5 \times 0.4597 = 0.22985\text{ psia}$$

- **Partial pressure of dry air ( $p_{da}$ );** from Equation (7), with  $p = 14.696\text{ psia}$  and  $p_w = 0.22985\text{ psia}$ :

$$p_{da} = p - p_w = 14.696 - 0.22985 = 14.26615\text{ psia}$$

- **Humidity ratio ( $W$ );** from Equation (13), with  $p = 14.496\text{ psia}$  and  $p_w = 14.496\text{ psia}$  and  $p_w = 0.22985\text{ psia}$ :

$$W = 0.621945 \frac{p_w}{p - p_w} = 0.621945 \frac{0.22985}{14.696 - 0.22985}$$

$$W = 0.01\text{ lb}_w/\text{lb}_{da}$$

- **Specific volume ( $v$ );** from Equation (22), with  $t = 77\text{ }^{\circ}\text{F}$ ,  $W = 0.01\text{ lb}_w/\text{lb}_{da}$  and  $p = 14.496\text{ psia}$ :

$$v = 0.370486(t + 459.67)(1 + 1.607858W)/p$$

$$v = 0.370486(77 + 459.67)(1 + 1.607858 \times 0.01)/14.496$$

$$v = 13.7469\text{ ft}^3/\text{lb}_{da}$$

- **Dew-point temperature ( $t_d$ )**; from Equation (27):

$$t_d = C_{14} + C_{15}\alpha + C_{16}\alpha^2 + C_{17}\alpha^3 + C_{18}p_w^{0.1984}$$

From  $p_w = 0.22985$  psia, we find

$$\alpha = \ln p_w = \ln (0.22985) = -1.470$$

Substitute the constants  $C$ ,  $p_w$ , and  $\alpha$ :

$$t_d = C_{14} + C_{15}(-1.470) + C_{16}(-1.470)^2 + C_{17}(-1.470)^3 + C_{18}(-1.470)^{0.1984}$$

$$t_d = 57.1 \text{ }^{\circ}\text{F}$$

- **Enthalpy of moist air ( $h$ )**; from Equation (32):

$$h = 0.240t + W(1061 + 0.444t)$$

with  $t = 77 \text{ }^{\circ}\text{F}$  and  $W = 0.01 \text{ lb}_w/\text{lb}_{da}$ :

$$h = 0.240 \times 77 + 0.01(1061 + 0.444 \times 77)$$

$$h = 29.427 \text{ Btu/lb}_{da}$$

## 1.6 Psychrometric Chart

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A psychrometric chart graphically represents the thermodynamic properties of moist air. Each chart typically applies to a specific constant pressure. Chart formats vary by source or manufacturer.

ASHRAE provides multiple psychrometric charts for sea level and for altitudes of 750 m (2500 ft), 1500 m (5000 ft), and 2250 m (7500 ft). Charts are also available for various temperature intervals, such as

- **Normal temperature range:** 0–50°C (32–120°F)
- **Low temperature range:** -40–10°C (-40–50°F)
- **High temperature range:** 10–120°C (60–250°F)
- **Very high temperature range:** 100–200°C (400–600°F)

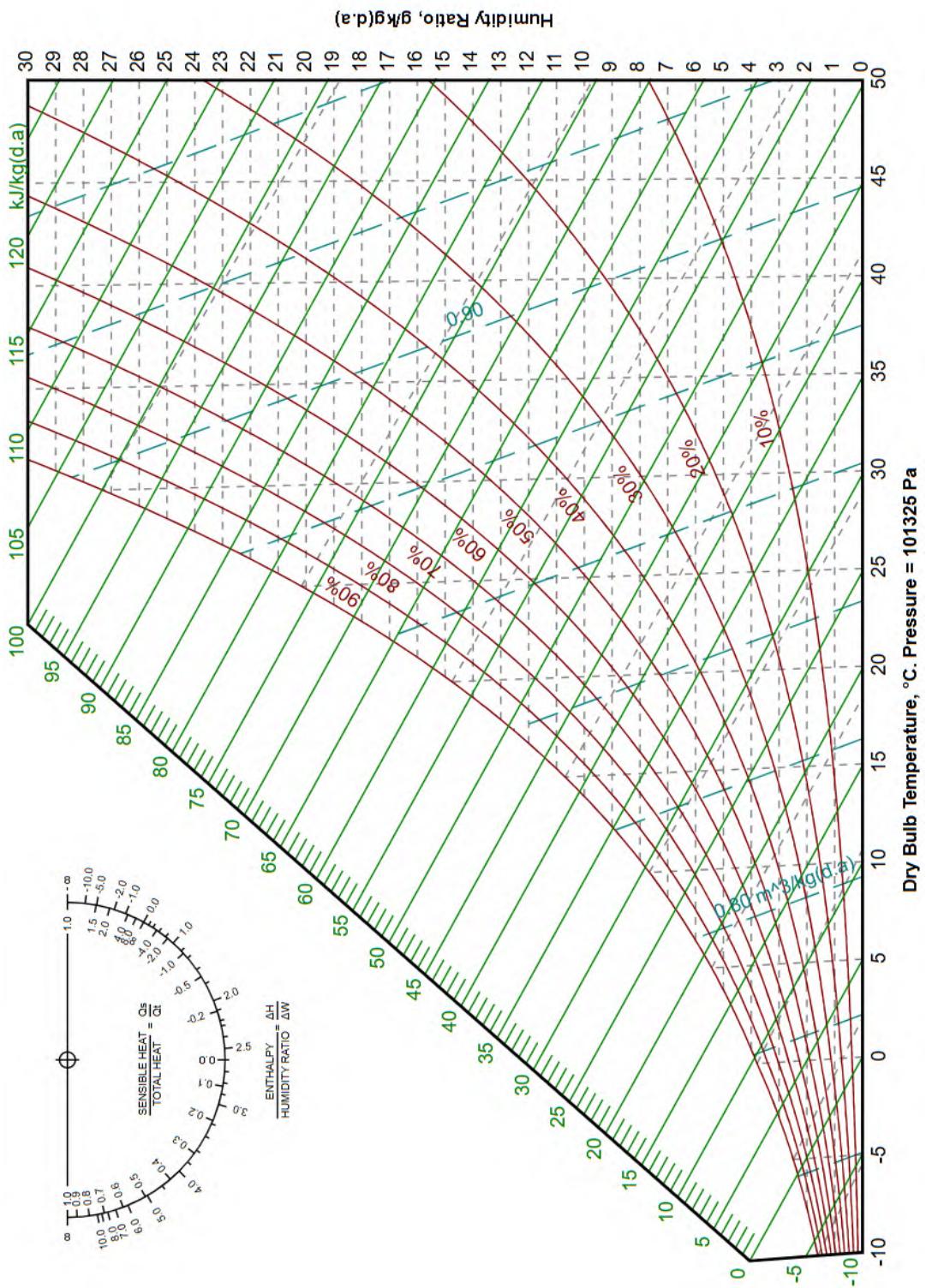
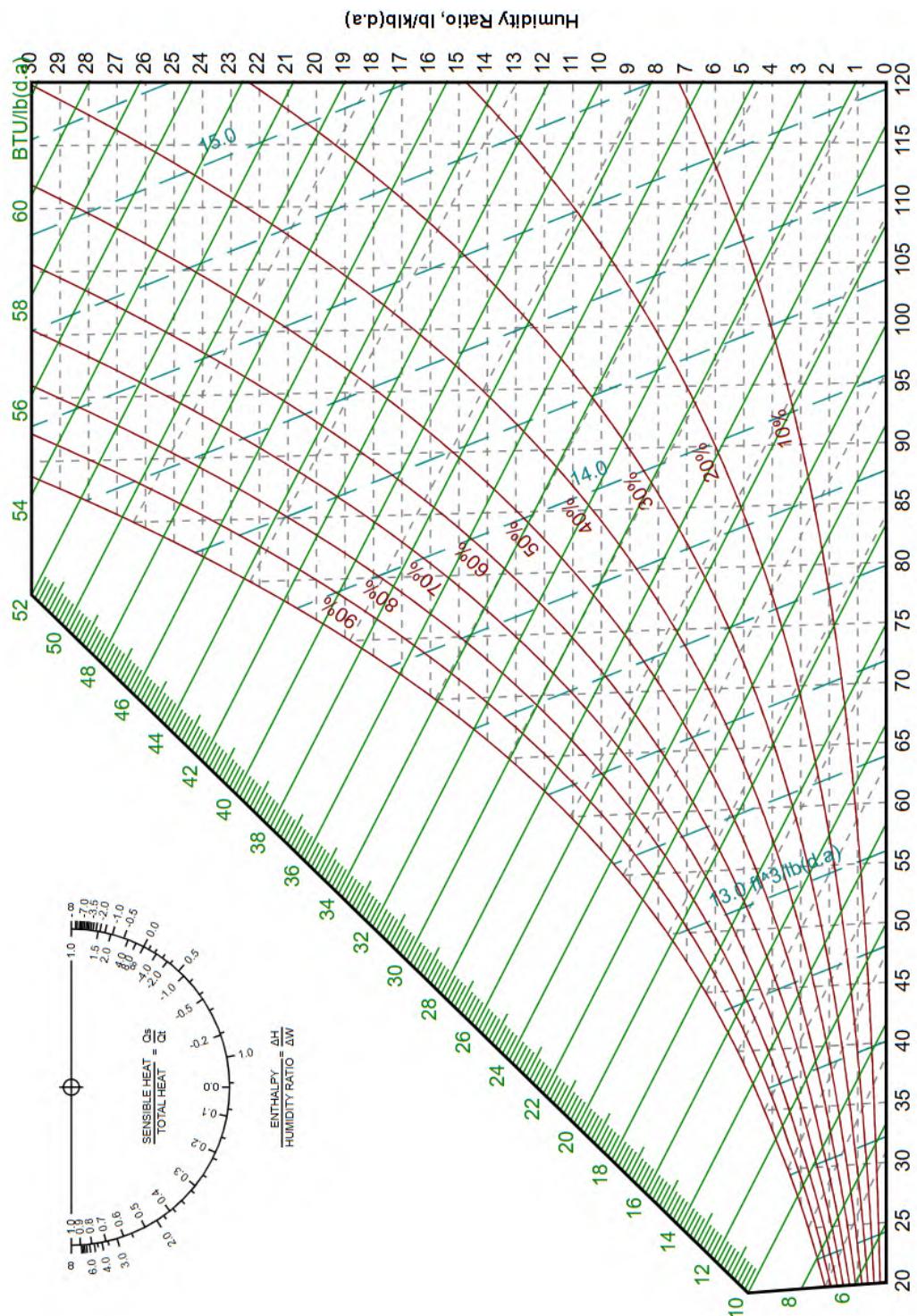
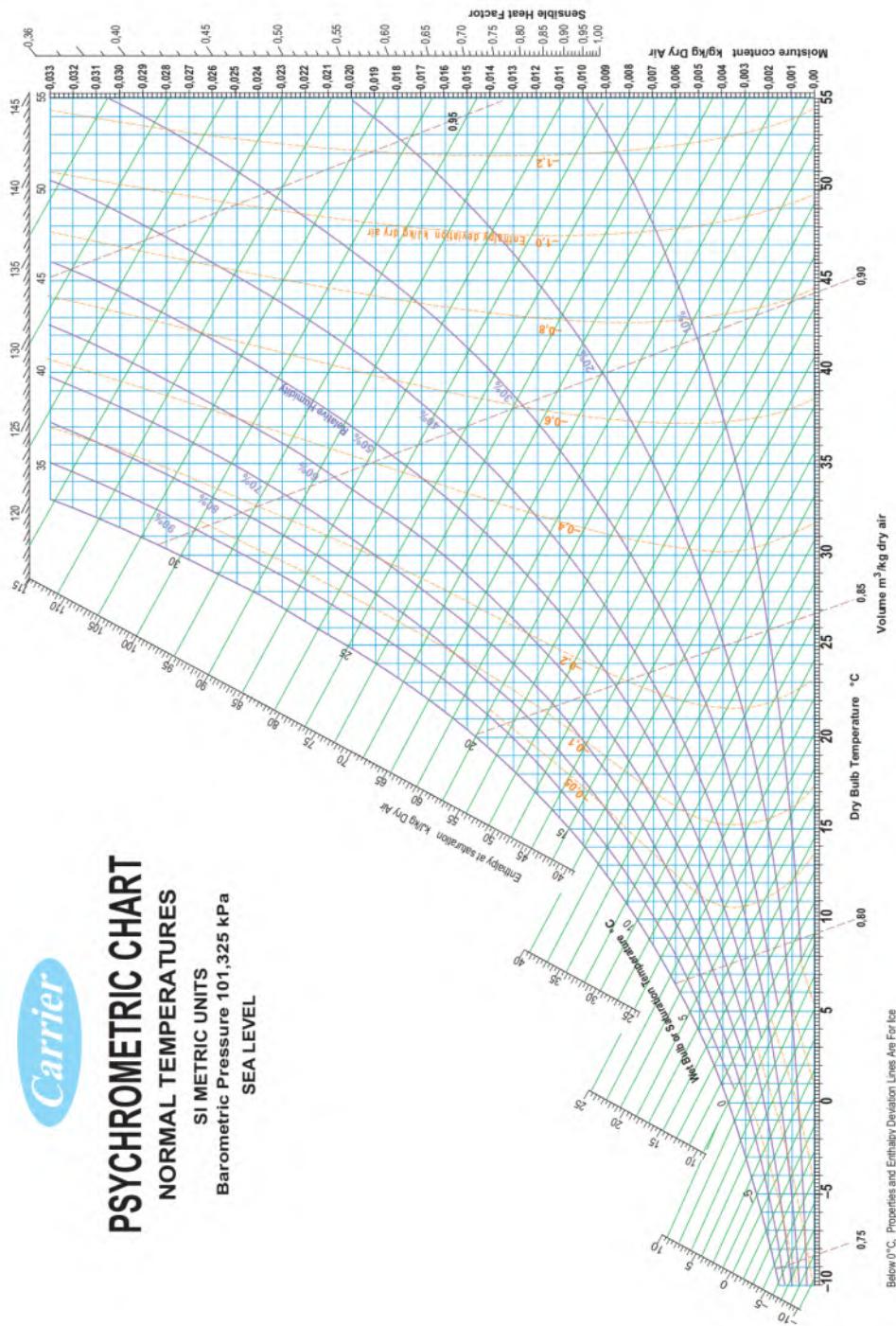


Fig. 1.3a Psychrometric Chart in SI units by FlyCarpet Inc [www.flycarpet.net](http://www.flycarpet.net)



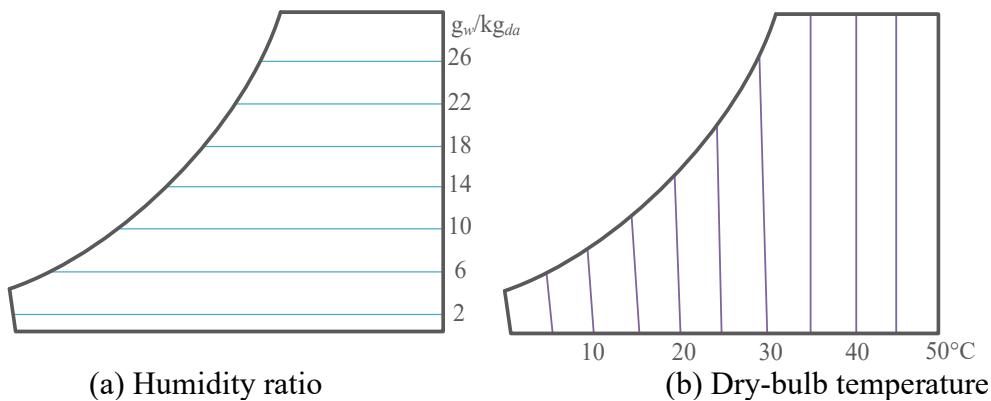
**Fig. 1.3b** Psychrometric Chart in IP units by FlyCarpet Inc [www.flycarpet.net](http://www.flycarpet.net)



**Fig. 1.4** Psychrometric Chart in SI Units of Carrier

Source: Carrier (2022)

The most commonly used psychrometric chart in air-conditioning applications is the standard sea-level temperature chart, as shown in [Figure 1.3](#). However, if the pressure differs from the chart value, equations from the moist air properties section are used instead. In [Figure 1.3](#), the right vertical axis represents humidity ratio ( $W$ ), ranging from 0 to 30  $\text{g}_w/\text{kg}_{da}$  (0-0.03  $\text{lb}_w/\text{lb}_{da}$ ). The horizontal axis shows dry-bulb temperature lines, which are slightly sloped leftward rather than vertical, as illustrated in [Figure 1.5](#).

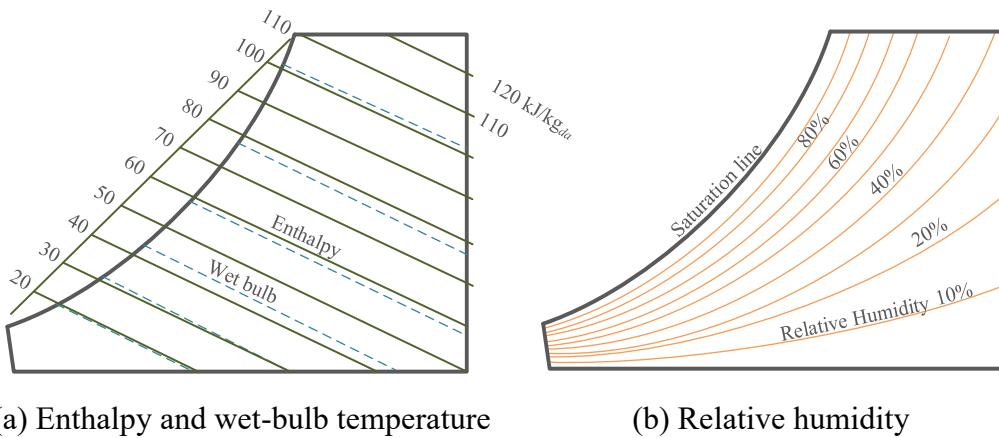


**Fig. 1.5** Humidity Ratio and Dry-Bulb Temperature Lines in SI Units

**Enthalpy** ( $h$ ) appears as diagonal lines across the psychrometric chart that are parallel. **Wet-bulb temperature lines** ( $t^*$ ) also slope diagonally in generally the same direction as enthalpy lines but are not parallel, as shown in [Figure 1.6](#). In the Carrier psychrometric chart, enthalpy readings differ from the ASHRAE chart. Carrier's chart shows saturated air enthalpy, which is not the actual value under typical conditions. Thus, enthalpy lines run parallel to wet-bulb temperature lines throughout, as illustrated in [Figure 1.4](#).

Carrier's saturated air enthalpy values deviate from actual values by no more than 2%. For greater accuracy, enthalpy values are corrected using the enthalpy deviation, shown as a vertical curve labeled "Enthalpy Deviation" in [Figure 1.4](#). For example, at 35°C dry-bulb temperature and 10% RH, the enthalpy from [Figure 1.4](#) is 44.5  $\text{kJ}/\text{kg}_{da}$ , with an enthalpy deviation of approximately  $-0.52 \text{ kJ}/\text{kg}_{da}$  (between -0.4 and -0.6). Thus, the actual enthalpy is  $44.5 - 0.52 = 43.98 \text{ kJ}/\text{kg}_{da}$ .

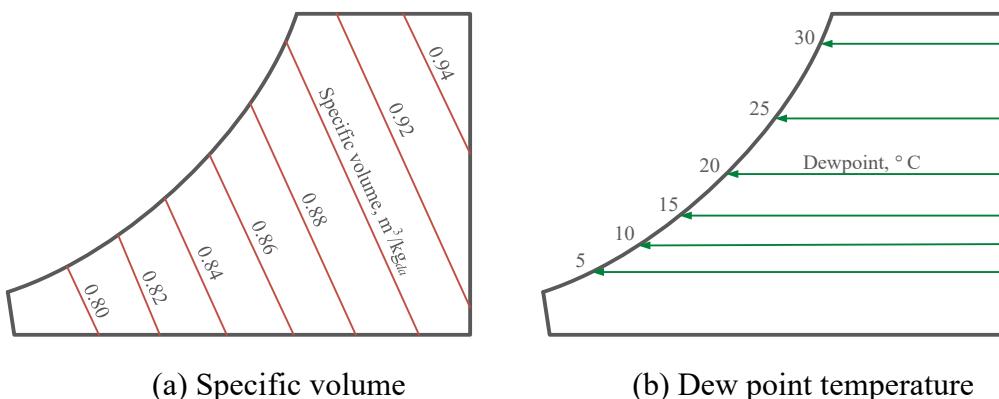
**Relative humidity** appears as curved lines spaced at 10% intervals. The saturation line, representing 100% RH, marks the boundary of saturated air. The bottom horizontal line of the chart represents conditions where both humidity ratio and relative humidity are zero (dry air), as shown in [Figure 1.6b](#).



**Fig. 1.6** Enthalpy, Wet-Bulb Temperature, and Relative Humidity Curves in SI Units

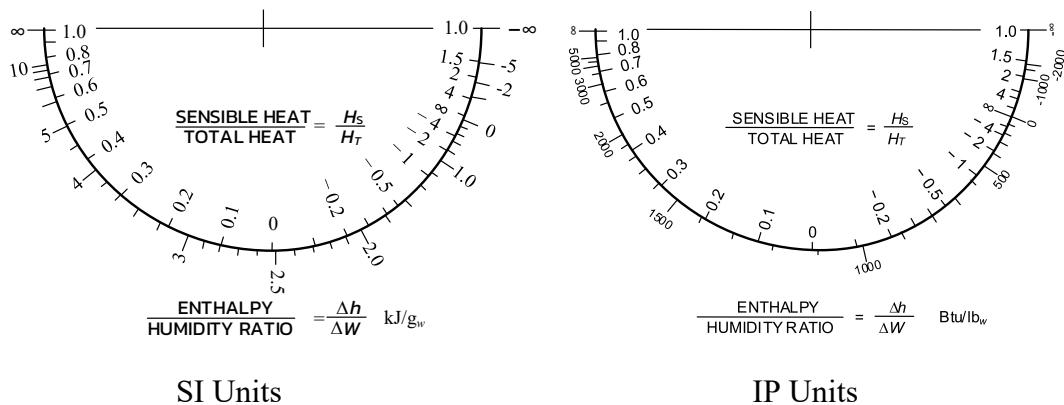
**Specific volume** lines are shown as diagonal lines, approximately parallel, as illustrated in [Figure 1.7a](#). For the dew point temperature, there is no dedicated line on the chart. Instead, it is determined by drawing a horizontal line from the given state point to intersect the saturation curve and reading the corresponding temperature, as shown in [Figure 1.7b](#). On the saturation curve, the temperatures represent the dry-bulb, wet-bulb, and dew point temperatures, as they are equal at saturation. This simplifies value reading from the chart.

The narrow region above the saturation curve is called the fog region. This area represents saturated moist air mixed with suspended liquid water in thermal equilibrium. In this region, constant-temperature lines follow extended wet-bulb lines.



**Fig. 1.7** Specific Volume Lines and Dew Point Temperature in SI Units

In the psychrometric chart, there is a semi-circular protractor located at the top left corner. The inner scale represents the Sensible Heat Ratio (SHR), while the outer scale shows the ratio of enthalpy difference to humidity ratio difference ( $\Delta h/\Delta W$ ), expressed in units of kJ/g (Btu/lb<sub>w</sub>), as illustrated in [Figure 1.8](#).



**Fig. 1.8** Example of SHR and  $\Delta h/\Delta W$  Protractor on a Psychrometric Chart in SI Units

Source: ASHRAE Handbook (2021)

These protractors serve as references for process lines on the psychrometric chart. Any process with the same Sensible Heat Ratio (SHR) or the same  $\Delta h/\Delta W$  will have parallel process lines. This functionality is particularly useful for analyzing processes in air conditioning, as demonstrated by examples in later sections.

The Carrier chart shows only the Sensible Heat Factor (SHF) scale on the right side of the chart. The reference point is the "Alignment Circle", corresponding to a dry-bulb temperature of 24°C and 50% relative humidity, as shown in [Figure 1.4](#).

**Example 1.2****SI Units**

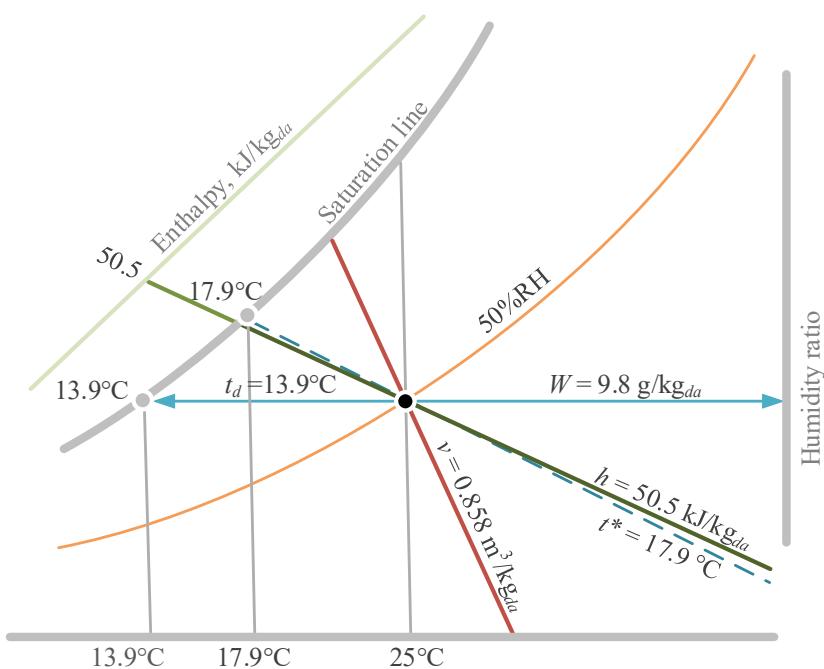
Based on Example 1.1, the air has a dry-bulb temperature of  $25^\circ\text{C}$  at 101.325 kPa, with 50% relative humidity ( $\phi = 0.5$ ). Determine the moist air thermodynamic properties.

Given:  $t = 25^\circ\text{C}$  (298.15 K),  $\phi = 0.5$ , and  $p = 101.325$  kPa.

Required: The remaining moist air properties.

Solution: Based on the ASHRAE psychrometric chart at 101.325 kPa and comparison with calculated results, the following properties are obtained:

Properties	From Equations	From chart
Humidity ratio, $W(\text{kg}_w/\text{kg}_{da})$	0.01	0.0098
Specific Volume, $v (\text{m}^3/\text{kg}_{da})$	0.8582	0.858
Dew-point temperature, $t_d$ ( $^\circ\text{C}$ )	13.89	13.87
Enthalpy, $h$ (kJ/kg <sub>da</sub> )	50.614	50.5
Wet bulb temperature, $t^*$ ( $^\circ\text{C}$ )	18.00	17.9



**Fig. 1.9(SI)** Example of Reading Values from a Psychrometric Chart

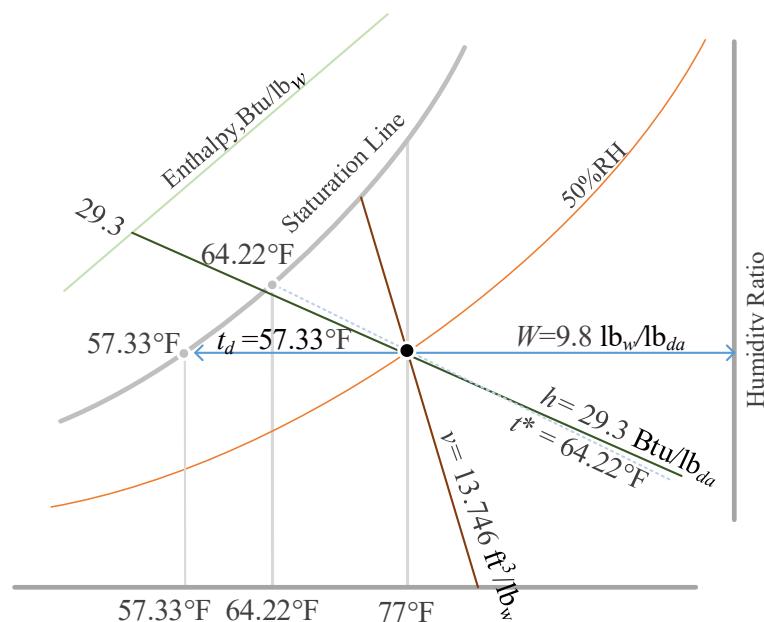
**Example 1.2** Based on Example 1.1, the air has a dry-bulb temperature of 77°F at 14.696 psia, with 50% relative humidity ( $\phi = 0.5$ ). Determine the moist air thermodynamic properties.

Given:  $t_1 = 77^\circ\text{F}$  (536.67°R),  $\phi = 0.5$ , and  $p = 14.696$  psia.

Required: The remaining moist air properties.

**Solution:** Based on the ASHRAE psychrometric chart at 14.696 psia and comparison with calculated results, the following properties are obtained:

Properties	From Equations	From chart
Humidity ratio, $W(\text{lb}_w/\text{lb}_{da})$	0.01	0.0098
Specific Volume, $v(\text{ft}^3/\text{lb}_{da})$	13.7469	13.746
Dew-point temperature, $t_d$ (°F)	57.33	57.33
Enthalpy, $h$ (Btu/lb <sub>da</sub> )	29.427	29.3
Wet bulb temperature, $t^*$ (°F)	64.40	64.22



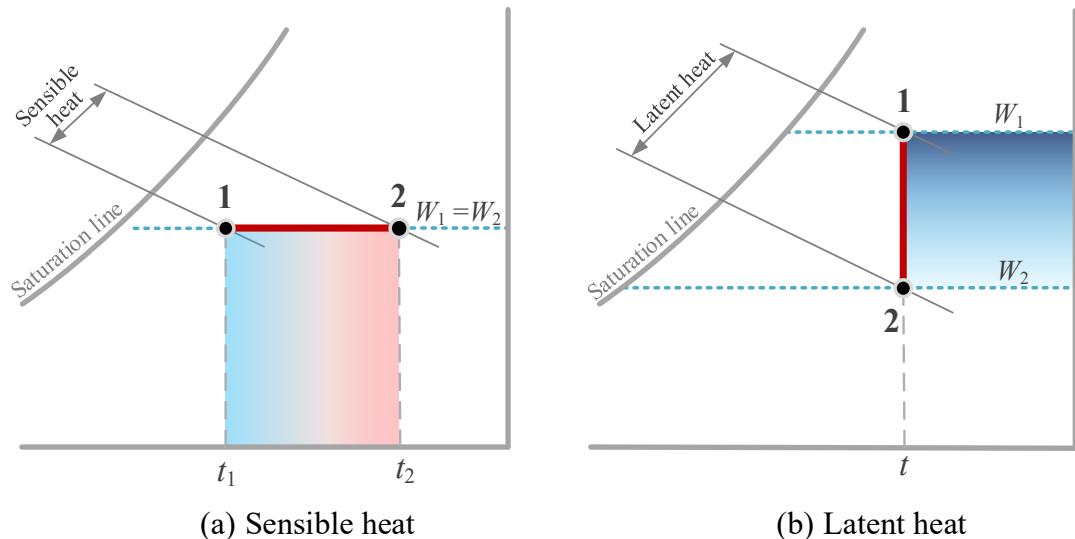
**Fig. 1.9(IP)** Example of Reading Values from a Psychrometric Chart

## 1.7 Typical Air-Conditioning Process

### 1.7.1 Sensible and Latent Heat

According to the thermodynamic definition, sensible heat is the heat exchange that changes temperature without a phase change, while latent heat is the heat exchange that causes a phase change at constant temperature. In air-conditioning processes, both sensible and latent heat are involved. Sensible heat occurs with a change in air temperature, while latent heat involves the phase change of water vapor, such as condensation or evaporation.

When analyzing processes on the psychrometric chart, sensible heat represents the heat transfer during the process from State 1 to State 2 or from State 2 to State 1, as shown in [Figure 1.10\(a\)](#). Meanwhile, latent heat represents the heat transfer during the process from State 1 to State 2 or from State 2 to State 1, as illustrated in [Figure 1.10\(b\)](#).



**Fig. 1.10** Sensible Heat and Latent Heat on the Psychrometric Chart

**Sensible heat of moist air:** Based on the first law of thermodynamics, the sensible heat of moist air is expressed as

$$q_s = \dot{m}_{da} \Delta h \quad \text{SI and IP} \quad (37)$$

Since sensible heat in moist air is the sum of the sensible heat of dry air and the sensible heat of water vapor, the sensible heat equation can therefore be written as follows:

$$q_s = \dot{m}_{da} \Delta h = c_{pda} \dot{m}_{da} \Delta t + c_{pw} \dot{m}_w \Delta t \quad \text{SI and IP} \quad (38)$$

where  $q_s$  = sensible heat of moist air, kW (Btu/hr)  
 $c_{pda}$  = specific heat capacity of dry air, 1.006 kJ/kg·K (0.24 Btu/lb<sub>da</sub>·°F)  
 $c_{pw}$  = specific heat of water vapor, 1.86 kJ/kg·K (0.444 Btu/lb<sub>w</sub>·°F)  
 $\dot{m}_{da}$  = mass flow rate of dry air, kg/s (lb<sub>da</sub>/hr)  
 $\dot{m}_w$  = mass flow rate of water vapor in air, kg/s (lb<sub>w</sub>/hr)  
 $t$  = air temperature, °C (°F)

By substituting  $c_p$  and  $\dot{m}_w = \dot{m}_{da} W$ , the **sensible heat of moist air** is calculated as follows:

$$q_s = \dot{m}_{da} (1.006 + 1.86W) \Delta t \quad \text{SI} \quad (39)$$

$$q_s = \dot{m}_{da} (0.24 + 0.444W) \Delta t \quad \text{IP} \quad (39)$$

The expression  $1.006 + 1.86W$  in SI units and  $0.24 + 0.444W$  in IP units is commonly known as the specific heat capacity of moist air ( $c_s$ ) (Don and Marylee, 2019)

**Latent heat of moist air** refers to the energy associated with the evaporation of water into air or the condensation of water vapor. This quantity may be determined using an approach analogous to Equation (37) or by independently analyzing the water vapor component, as shown below.

$$q_l = \dot{m}_w h_{g1} - \dot{m}_w h_{g2} \quad \text{SI and IP} \quad (40)$$

where  $q_l$  = latent heat of moist air, kW (Btu/hr)  
 $h_g$  = enthalpy of water vapor, kJ/kg<sub>da</sub> (Btu/lb<sub>da</sub>)

where  $\dot{m}_w = \dot{m}_{da} W$ . Substituting  $h_g \approx 2501 + 1.86t$  (1061 + 0.444t, in IP units):

$$q_l = \dot{m}_{da} [W_1(2501 + 1.86t_1) - W_2(2501 + 1.86t_2)] \quad \text{SI}$$

$$q_l = \dot{m}_{da} [W_1(1061 + 0.444t_1) - W_2(1061 + 0.444t_2)] \quad \text{IP}$$

$$q_l = \dot{m}_{da} [2501 \Delta W + 1.86(t_1 W_1 - t_2 W_2)] \quad \text{SI}$$

$$q_l = \dot{m}_{da} [1061 \Delta W + 0.444(t_1 W_1 - t_2 W_2)] \quad \text{IP}$$

where  $\Delta W = W_1 - W_2$

From the equation, the term  $1.86(t_1W_1 - t_2W_2)$  is approximated by  $1.86t_m\Delta W$ , where  $t_m$  is the average temperature during the process. **The latent heat of moist air** is expressed as follows:

$$q_l = \dot{m}_{da}\Delta W [2501 + 1.86t_m] \quad \text{SI} \quad (41)$$

$$q_l = \dot{m}_{da}\Delta W [1061 + 0.444t_m] \quad \text{IP} \quad (41)$$

**Airflow rate:** In air-conditioning applications, the airflow rate is typically expressed in volumetric terms, such as cubic meters per second ( $\text{m}^3/\text{s}$ ) or cubic feet per minute (cfm). The volumetric airflow rate of air is given by the following expression:

$$\dot{m}_{da} = Q/v \quad \text{SI} \quad (42)$$

$$\dot{m}_{da} = 60Q/v \quad \text{IP} \quad (42)$$

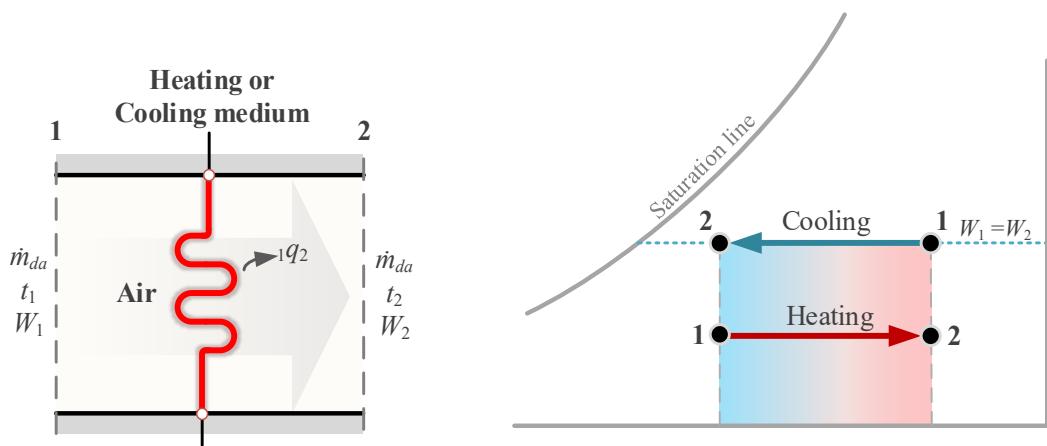
where  $Q$  = volumetric airflow rate of air,  $\text{m}^3/\text{s}$  (cfm)  
 $v$  = specific volume of air,  $\text{m}^3/\text{kg}_{da}$  ( $\text{ft}^3/\text{lb}_{da}$ )  
60 = conversion factor from seconds to minutes  
 $\dot{m}_{da}$  = mass flow rate of dry air,  $\text{kg}/\text{s}$  ( $\text{lb}_{da}/\text{hr}$ )

**Note:** When calculating the mass flow rate of air ( $\dot{m}_{da}$ ), the volumetric flow rate and the specific volume ( $v$ ) must be taken at the same measurement point.

**Air-conditioning process analysis** can be performed in two ways: by analyzing the properties of moist air or by separating the analysis of water vapor and dry air. In general, the moist air approach is more commonly used, with calculations based on the mass of dry air, because most available property data for moist air are expressed per unit mass of dry air. These properties can be found in psychrometric charts or in the ASHRAE Handbook—Fundamentals. Moreover, in a typical air-conditioning process, the mass of dry air tends to remain constant, whereas the mass of moist air may vary if water vapor is added to or removed from the system. It is worth noting that certain handbooks indicate the quantity of water vapor involved in typical air-conditioning processes is sufficiently small to be neglected, thereby simplifying calculations.

### 1.7.2 Moist Air Sensible Heating or Cooling

In air-conditioning systems, heating processes are common, such as electric, steam, or hot oil coils, or heat transfer at the condenser coil of the air-conditioning unit. These processes involve the addition of heat while the humidity ratio remains constant. Conversely, coils commonly known as dry coils undergo cooling processes that maintain a constant humidity ratio. These processes occur at constant pressure, as illustrated in [Figure 1.11](#).



**Fig. 1.11** Schematic for Heating or Cooling Process at Constant Humidity Ratio

*Mass balance equations:*

$$\text{dry air:} \quad \dot{m}_{da1} = \dot{m}_{da2} \quad \text{SI and IP} \quad (43)$$

$$\text{water vapor:} \quad \dot{m}_{da1}W_1 = \dot{m}_{da2}W_2 \quad \text{SI and IP} \quad (44)$$

*First law of thermodynamics:*

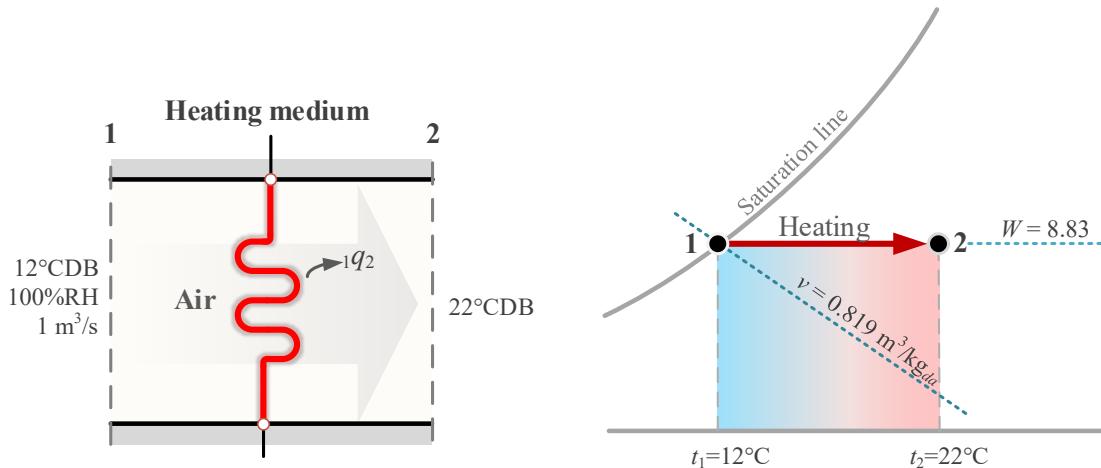
Based on the properties of moist air:

$$1q_2 = \dot{m}_{da} (h_2 - h_1) \quad \text{SI and IP} \quad (45)$$

Equation (45) represents sensible heat only; therefore, its result is equivalent to that of Equation (39).

**Example 1.3****SI Units**

Saturated air at 12°C enters a heating coil at an airflow rate of 1 m<sup>3</sup>/s and exits at 22°C. Determine the heat transferred to the air by the heating coil.



**Fig. 1.12(SI)** Schematic Solution for Example 1.3

Given:  $t_1 = 12^\circ\text{C}$ , saturated air (100% RH),  $Q = 1 \text{ m}^3/\text{s}$ ,  $t_2 = 22^\circ\text{C}$ ,  $p = 101.325 \text{ kPa}$ .

Required:  $q_2$ .

Solution: Determine  $\dot{m}_{da}$  from the air properties at State 1 (since the airflow rate at State 1 is given).

- The saturation pressure ( $p_{ws}$ ) is calculated by substituting  $T = 273.15 + 12 = 285.15 \text{ K}$  into Equation (3):

$$\ln p_{ws} = C_8/T + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

$$p_{ws} = 1417.8 \text{ Pa} \text{ (equal } p_w \text{ since the air is saturated)}$$

- The humidity ratio ( $W$ ) is calculated from Equation (13).

$$W = 0.621945 \frac{1.4178}{101.325 - 1.4178}$$

$$= 0.00883 \text{ kg}_w/\text{kg}_{da}$$

- The specific volume ( $v$ ) is calculated by substituting  $t_1$ ,  $W$ , and  $p$  into Equation (22):

$$v = 0.287042(t + 273.15)(1 + 1.607858W)/p$$

$$v = 0.287042(12 + 273.15)(1 + 1.607858 \times 0.00883)/101.325$$

$$v = 0.81926 \text{ m}^3/\text{kg}_{da}$$

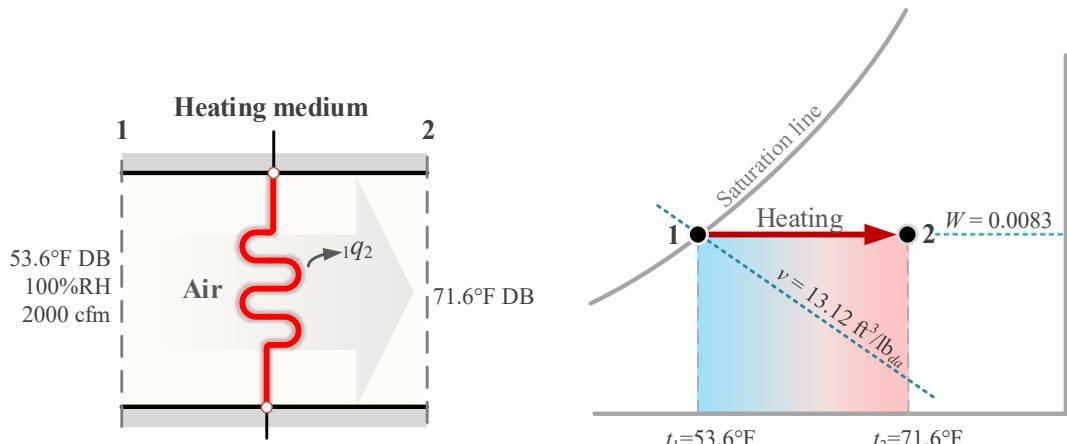
- The dry air mass flow rate ( $\dot{m}_{da}$ ) is calculated from Equation (42).

$$\begin{aligned}\dot{m}_{da} &= Q/v \\ &= 1/0.81926 \\ &= 1.2206 \text{ kg/s}\end{aligned}$$

- All values are substituted into Equation (39) to calculate  $1q_2$ :

$$\begin{aligned}1q_2 &= \dot{m}_{da} (1.006 + 1.86W)\Delta t \\ &= 1.2206(1.006 + 1.86 \times 0.00883)(22-12) \\ &= 12.48 \text{ kW}\end{aligned}$$

**Example 1.3** Saturated air at 53.6°F enters a heating coil at an airflow rate of 2000 cfm and exits at 71.6°F. Determine the heat transferred to the air by the heating coil.



**Fig. 1.12(IP)** Schematic Solution for Example 1.3

Given:  $t_1 = 53.6^\circ\text{F}$ , saturated air,  $Q = 2000 \text{ cfm}$ ,  $t_2 = 71.6^\circ\text{F}$ ,  $p = 14.696 \text{ psia}$

Required:  $1q_2$ .

Solution: Determine  $\dot{m}_{da}$  from the air properties at State 1 (since the airflow rate at State 1 is given).

- The saturation pressure ( $p_{ws}$ ) is calculated by substituting  $T = 53.6 + 459.7 = 513.3^{\circ}\text{R}$  into Equation (3):

$$\ln p_{ws} = C_8/T + C_9 + C_{10}T + C_{11}T^2 + C_{12}T^3 + C_{13} \ln T$$

$$p_{ws} = 0.206 \text{ psia} \text{ (equal } p_w \text{ since the air is saturated)}$$

- The humidity ratio ( $W$ ) is calculated from Equation (13).

$$W = 0.621945 \frac{0.206}{14.696 - 0.206}$$

$$= 0.00883 \text{ lb}_w/\text{lb}_{da}$$

- The specific volume ( $v$ ) is calculated by substituting  $t_1$ ,  $W$ , and  $p$  into Equation (22):

$$v = 0.370486(t + 459.67)(1 + 1.607858W)/p$$

$$v = 0.370486(53.6 + 459.67)(1 + 1.607858 \times 0.00883)/14.696$$

$$v = 13.12 \text{ ft}^3/\text{lb}_{da}$$

- The dry air mass flow rate ( $\dot{m}_{da}$ ) is calculated from Equation (42).

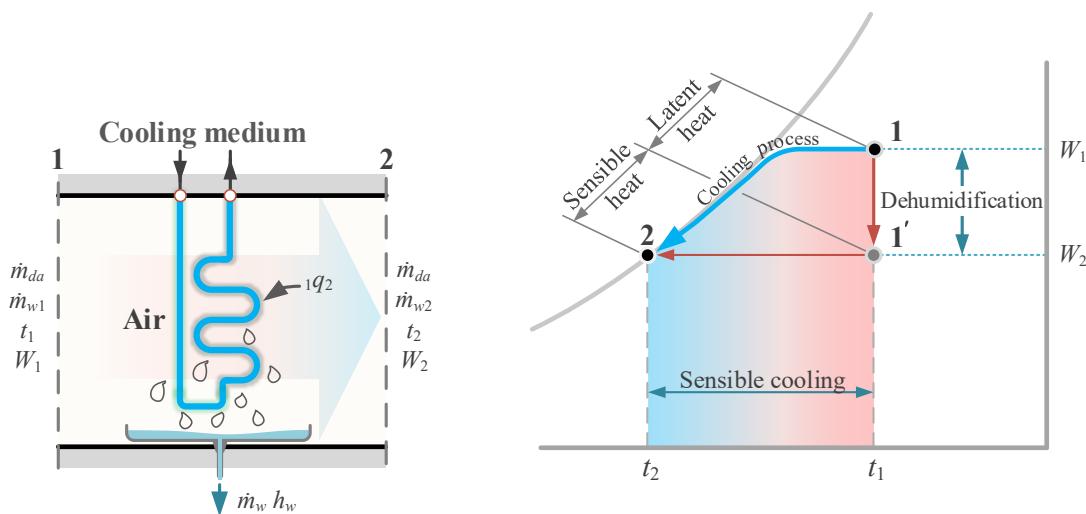
$$\begin{aligned} \dot{m}_{da} &= 60Q/v \\ &= 60(2000/13.12) \\ &= 9146 \text{ lb}_{da}/\text{hr} \end{aligned}$$

- All values are substituted into Equation (39) to calculate  ${}_1q_2$ :

$$\begin{aligned} {}_1q_2 &= \dot{m}_{da} (0.24 + 0.444W)\Delta t \\ &= 9146(0.24 + 0.444 \times 0.00883)(71.6 - 53.6) \\ &= 40143 \text{ Btu/h} \end{aligned}$$

### 1.7.3 Moist Air Cooling and Dehumidification

In a typical air-conditioning process, cooling occurs at the cooling coil of the air conditioner. As air flows through the coil, its temperature decreases until it drops below the initial dew point. At this state, water vapor condenses into droplets, gradually changing the air state. Condensation continues until the final temperature is reached, and the condensate exits the coil at the same temperature as the final state. This process is known as cooling and dehumidification, as illustrated in [Figure 1.13](#).



**Fig. 1.13** Schematic for Moist Air Cooling and Dehumidification

*Mass balance equations:*

$$\text{dry air:} \quad \dot{m}_{da1} = \dot{m}_{da2} = \dot{m}_{da} \quad \text{SI and IP} \quad (46)$$

$$\text{water vapor:} \quad \dot{m}_{da1}W_1 = \dot{m}_{da2}W_2 + \dot{m}_w \quad \text{SI and IP} \quad (47)$$

*First law of thermodynamics:*

Based on the properties of moist air:

$$\begin{aligned} \dot{m}_{da}h_1 &= \dot{m}_{da}h_2 + {}_1q_2 + \dot{m}_w h_w \\ {}_1q_2 &= \dot{m}_{da}[(h_1 - h_2) - (W_1 - W_2)h_w] \end{aligned} \quad \text{SI and IP} \quad (48)$$

As shown in [Figure 1.13](#),  ${}_1q_2$  represents the total heat, including latent and sensible heat of the dry air, minus the energy of the condensed water. The sum is expressed by the following equation.

$$_1q_2 = q_s + q_l - \dot{m}_w h_w \quad \text{SI and IP} \quad (49)$$

By substituting Equations (39), (41), and  $h_w \approx 4.186t$  (or  $t - 32$  in IP units) into Equations (49)

$$_1q_2 = \dot{m}_{da}[(1.006 + 1.86W_m)\Delta t + (2501 + 1.86t_m)\Delta W - 4.186t_2\Delta W] \quad \text{SI} \quad (50)$$

$$_1q_2 = \dot{m}_{da}[(0.24 + 0.444W_m)\Delta t + (1061 + 0.444t_m)\Delta W - (t_2 - 32)\Delta W] \quad \text{IP} \quad (50)$$

where  $\Delta t = t_1 - t_2$ ,  $\Delta W = W_1 - W_2$ , and  $t_m$  and  $W_m$  are the average temperature and average humidity ratio during the process.

### Assumptions for air-conditioning systems:

- 1) In typical air-conditioning processes,  $t_m \approx 24^\circ\text{C}$  ( $75^\circ\text{F}$ ), 55% RH and  $W_m \approx 0.01$
- 2) At State 2,  $t_2 \approx 10^\circ\text{C}$  ( $50^\circ\text{F}$ )

Based on these assumptions, substitute the values into Equation (50) to determine the total heat for the cooling process.

$$_1q_2 = \dot{m}_{da}[(1.006 + 1.86 \times 0.01)\Delta t + (2501 + 1.86 \times 24 - 4.186 \times 10)\Delta W]$$

$$_1q_2 = \dot{m}_{da}[(0.24 + 0.444 \times 0.01)\Delta t + (1061 + 0.444 \times 75 - (50 - 32) \times \Delta W)]$$

$$_1q_2 = \dot{m}_{da}(1.025\Delta t + 2504\Delta W) \quad \text{SI} \quad (51)$$

$$_1q_2 = \dot{m}_{da}(0.244\Delta t + 1076\Delta W) \quad \text{IP} \quad (51)$$

From the equation, the first term represents sensible heat, while the second term includes latent heat, minus the energy of the condensed water. For simplicity, this term is often referred to as latent heat.

**Note:** In typical air-conditioning systems, a portion of the airflow bypasses the cooling coil without contacting the coil surface. This bypass air causes deviation from the ideal process line shown in [Figure 1.13](#). This phenomenon is discussed in detail in later sections.

**Example 1.4****SI Units**

Air at  $30^\circ\text{C}$  and 50% RH enters a cooling coil at an airflow rate of  $5 \text{ m}^3/\text{s}$  and exits at  $10^\circ\text{C}$  under saturated conditions. Determine the heat transferred during the process and the condensate flow rate.

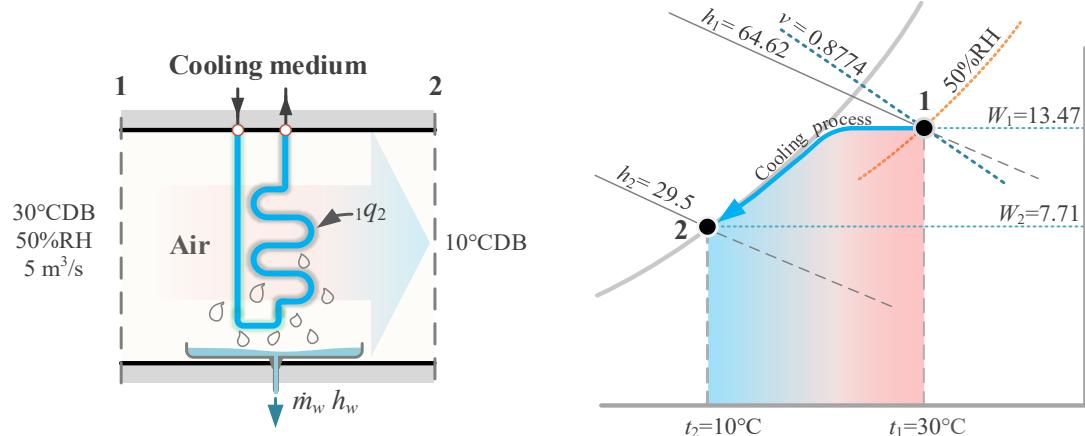
Given:  $t_1 = 30^\circ\text{C}$ , 50%RH,  $Q_1 = 5 \text{ m}^3/\text{s}$ ,  $t_2 = 10^\circ\text{C}$ , saturated air (100% RH),  $p = 101.325 \text{ kPa}$ .

Required:  $q_2$  and  $\dot{m}_w$ .

Solution: Use a spreadsheet to calculate air properties.

- From spreadsheet calculations, the following values are obtained:

$$v_1 = 0.8774 \text{ m}^3/\text{kg}_{da}, W_1 = 13.47 \text{ g/kg}_{da}, W_2 = 7.71 \text{ g/kg}_{da}, \\ h_1 = 64.62 \text{ kJ/kg}_{da}, h_2 = 29.5 \text{ kJ/kg}_{da}.$$



**Fig 1.14(SI)** Schematic Solution for Example 1.4

- The dry air mass flow rate ( $\dot{m}_{da}$ ) is calculated by substituting  $v_1$  into Equation (42).

$$\dot{m}_{da} = Q_1/v_1 = 5/0.8774 = 5.699 \text{ kg/s}$$

- From Equation (34),

$$h_{w2} = 4.186t = 4.186 \times 10 = 41.86 \text{ kJ/kg}_w$$

- By substituting all values into Equation (48), the total heat transfer for the process is

$$\begin{aligned} {}_1q_2 &= \dot{m}_{da} [(h_1 - h_2) - (W_1 - W_2)h_{w2}] \\ &= 5.699[(64.62 - 29.5) - (13.47 - 7.71)/1000 \times 41.86] \\ &= 198.8 \text{ kW} \end{aligned}$$

- From Equation (47), the condensate flow rate is

$$\begin{aligned} \dot{m}_w &= \dot{m}_{da}(W_1 - W_2) \\ \dot{m}_w &= 5.699 \times (13.47 - 7.71)/1000 \\ \dot{m}_w &= 0.0328 \text{ kg/s} \end{aligned}$$

**Example 1.4**
**IP Units**

Air at 86°F and 50% RH enters a cooling coil at an airflow rate of 10,000 cfm and exits at 50°F under saturated conditions. Determine the heat transferred during the process and the condensate flow rate.

Given:  $t_1 = 86^\circ\text{F}$ , 50% RH,  $Q_1 = 10000 \text{ cfm}$ ,  $t_2 = 50^\circ\text{F}$ , saturated air,  $p = 14.70 \text{ psia}$ .

Required:  ${}_1q_2$  and  $\dot{m}_w$

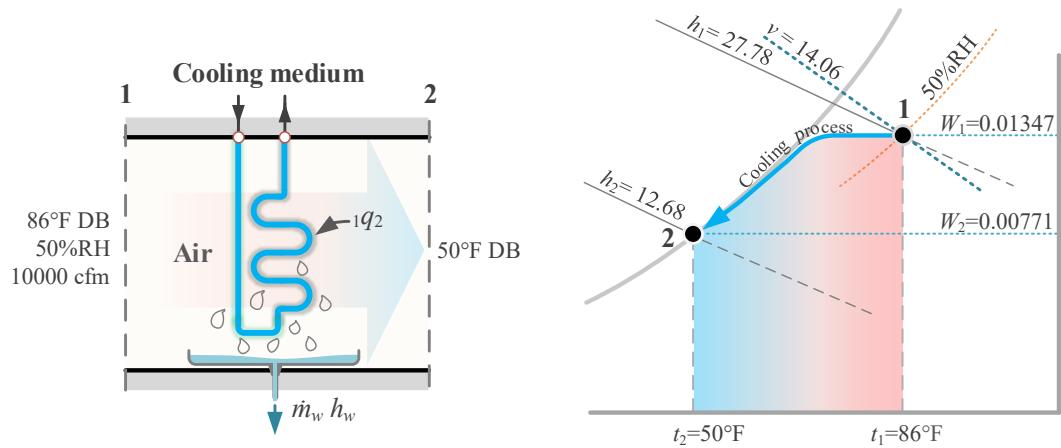
Solution: Use a spreadsheet to calculate air properties.

- From spreadsheet calculations, the following values are obtained:
 
$$\begin{aligned} v_1 &= 14.06 \text{ ft}^3/\text{lb}_{da}, W_1 = 0.01347 \text{ lb}_w/\text{lb}_{da}, W_2 = 0.00771 \text{ lb}_w/\text{lb}_{da}, \\ h_2 &= 35 \text{ Btu/lb}_{da}, h_2 = 20 \text{ Btu/lb}_{da}. \end{aligned}$$
- The dry air mass flow rate ( $\dot{m}_{da}$ ) is calculated by substituting  $v_1$  into Equation (42).

$$\dot{m}_{da} = 60Q_1/v_1 = 60 \times 10000/14.06 = 42674 \text{ lb}_{da}/\text{hr}$$

- From Equation (34).

$$h_{w2} = t - 32 = 50 - 32 = 18 \text{ Btu/lb}_w$$



**Fig 1.14(IP)** Schematic Solution for Example 1.4

- By substituting all values into Equation (48), the total heat transfer for the process is

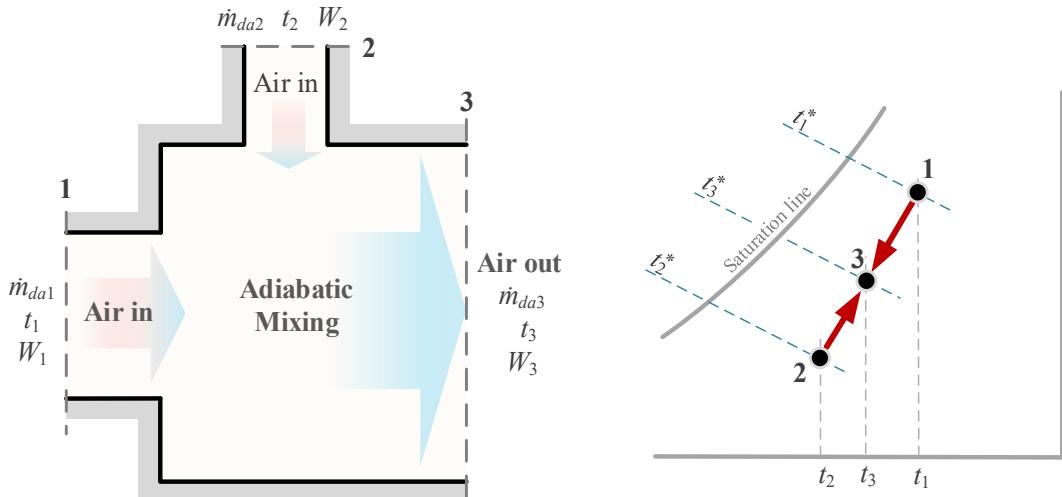
$$\begin{aligned}
 {}_1q_2 &= \dot{m}_{da} [(h_1 - h_2) - (W_1 - W_2)h_{w2}] \\
 &= 42674[(35 - 20) - (0.01347 - 0.00771)18] \\
 &= 635686 \text{ Btu/h}
 \end{aligned}$$

- From Equation (47), the condensate flow rate is

$$\begin{aligned}
 \dot{m}_w &= \dot{m}_{da}(W_1 - W_2) \\
 \dot{m}_w &= 42674 \times (0.01347 - 0.00771) \\
 \dot{m}_w &= 246 \text{ lb}_w/\text{hr}
 \end{aligned}$$

### 1.7.4 Adiabatic Mixing of Two Air Streams

In air-conditioning systems, air mixing typically occurs in the mixing or return air chamber of the air-handling unit (AHU), involving outdoor air and return air from the conditioned space. As illustrated in [Figure 1.15](#), air at State 1 mixes with air at State 2, forming mixed air at State 3.



**Fig. 1.15** Schematic for Adiabatic Mixing of Two Air-Streams

*Mass balance equations:*

$$\text{dry air: } \dot{m}_{da1} + \dot{m}_{da2} = \dot{m}_{da} \quad \text{SI and IP} \quad (52)$$

$$\text{water vapor: } \dot{m}_{da1}W_1 + \dot{m}_{da2}W_2 = \dot{m}_{da3}W_3 \quad \text{SI and IP} \quad (53)$$

*First law of thermodynamics:*

Based on moist air property analysis and assuming no heat loss, the following relationship is obtained:

$$\dot{m}_{da1}h_1 + \dot{m}_{da2}h_2 = \dot{m}_{da3}h_3 \quad \text{SI and IP} \quad (54a)$$

By substituting  $\dot{m}_{da3}$ :

$$\frac{h_2-h_3}{h_3-h_1} = \frac{W_2-W_3}{W_3-W_1} = \frac{\dot{m}_{da1}}{\dot{m}_{da2}} \quad \text{SI and IP} \quad (54b)$$

$$\text{or } \frac{h_2-h_3}{W_2-W_3} = \frac{h_3-h_1}{W_3-W_1} \quad \text{SI and IP} \quad (54c)$$

Equation (54) is in the form  $\Delta h/\Delta W$ , which corresponds to a straight line on the psychrometric chart's semicircular protractor. Therefore, the process lines from State 1 to State 3 and from State 2 to State 3 are parallel. Since both processes share State 3, States 1, 2, and 3 lie on the same straight line on the psychrometric chart, as illustrated in [Figure 1.15](#). Moreover, State 3 is always closer to the state with a higher mass flow rate.

Rearranging Equation (53) gives

$$W_3 = \frac{\dot{m}_{da1}W_1 + \dot{m}_{da2}W_2}{\dot{m}_{da3}} \quad \text{SI and IP} \quad (55)$$

Substituting the enthalpy of moist air from Equation (32) into Equation (54a) yields

$$\begin{aligned} \dot{m}_{da1}[(1.006t_1 + W_1(2501 + 1.86t))] + \dot{m}_{da2}[1.006t_2 + W_2(2501 + 1.86t)] = \\ \dot{m}_{da3}[1.006t_3 + W_3(2501 + 1.86t_3)] \end{aligned}$$

Rearranging the equation, the following expression is obtained:

$$t_3 = \frac{\dot{m}_{da1}(1.006 + 1.86W_1)t_1 + \dot{m}_{da2}(1.006 + 1.86W_2)t_2}{\dot{m}_{da3}(1.006 + 1.86W_3)} \quad \text{SI} \quad (56)$$

$$t_3 = \frac{\dot{m}_{da1}(0.24 + 0.444W_1)t_1 + \dot{m}_{da2}(0.24 + 0.444W_2)t_2}{\dot{m}_{da3}(0.24 + 0.444W_3)} \quad \text{IP} \quad (56)$$

From Equation (56),  $t_3$  can be determined by substituting  $W$ . However, in a typical air-conditioning process, the humidity ratio ( $W$ ) is relatively small. Thus, the term  $1.006 + 1.86W$  (or  $0.24 + 0.444W$ , in IP units) can be approximated and canceled out. Consequently, an approximate expression for  $t_3$  is

$$t_3 = \frac{\dot{m}_{da1}t_1 + \dot{m}_{da2}t_2}{\dot{m}_{da3}} \quad \text{SI and IP} \quad (57)$$

Once either the temperature  $t_3$  or humidity ratio  $W_3$  is known, other psychrometric properties at State 3 can be determined. However, such calculations are often complex and time-consuming. A more practical approach involves using the straight line connecting States 1 and 2 on the psychrometric chart. By drawing a horizontal line for  $W_3$  or a vertical line for  $t_3$  ( $W_3$  yields greater accuracy) until it intersects the line between States 1 and 2, State 3 can then be identified, as illustrated in [Figure 1.15](#). The remaining air property at State 3 can be directly read from the chart.

When States 1 and 2 are close, their specific volumes are nearly equal. By substituting  $\dot{m}_{da}$  from Equation (42) into Equations (55) and (57), **the approximate mixed-air temperature and humidity ratio** can be derived as follows:

$$t_3 = \frac{Q_1 t_1 + Q_2 t_2}{Q_3} \quad \text{and} \quad W_3 = \frac{Q_1 W_1 + Q_2 W_2}{Q_3} \quad \text{SI and IP} \quad (58)$$

where  $Q$  = volumetric flow rate of air,  $\text{m}^3/\text{s}$  (cfm)

$t$  = air temperature,  $^\circ\text{C}$  ( $^\circ\text{F}$ )

$W$  = humidity ratio,  $\text{kg}_w/\text{kg}_{da}$  ( $\text{lb}_w/\text{lb}_{da}$ )

### Example 1.5

#### SI Units

Outdoor air at  $35^\circ\text{C}$  dry-bulb and 60% RH at an airflow rate of  $720 \text{ m}^3/\text{h}$  is mixed with indoor air at  $25^\circ\text{C}$  dry-bulb and 50% RH at an airflow rate of  $5400 \text{ m}^3/\text{h}$  in the adiabatic mixing chamber of the air-handling unit (AHU). Determine the dry-bulb temperature and relative humidity of the mixed air.

Given:  $t_1 = 35^\circ\text{C}$ , 60%RH,  $Q_1 = 720/3600 = 0.2 \text{ m}^3/\text{s}$ ,  $t_2 = 25^\circ\text{C}$ , 50%RH,  $Q_2 = 5400/3600 = 1.5 \text{ m}^3/\text{s}$ ,  $p = 101.325 \text{ kPa}$ .

Required:  $t_2$  and  $\phi_2$

Solution: From spreadsheet calculations:  $v_1 = 0.9034 \text{ m}^3/\text{kg}_{da}$ ,  $W_1 = 21.7 \text{ g/kg}_{da}$ ,  $v_2 = 0.8582 \text{ m}^3/\text{kg}_{da}$ ,  $W_2 = 10.0 \text{ g/kg}_{da}$ .

- The dry air mass flow rate ( $\dot{m}_{da}$ ) is calculated by substituting  $v$  into Equation (42).

$$\dot{m}_{da1} = Q_1/v_1 = 0.2/0.9034 = 0.2214 \text{ kg/s}$$

$$\dot{m}_{da2} = Q_2/v_2 = 1.5/0.8582 = 1.7478 \text{ kg/s}$$

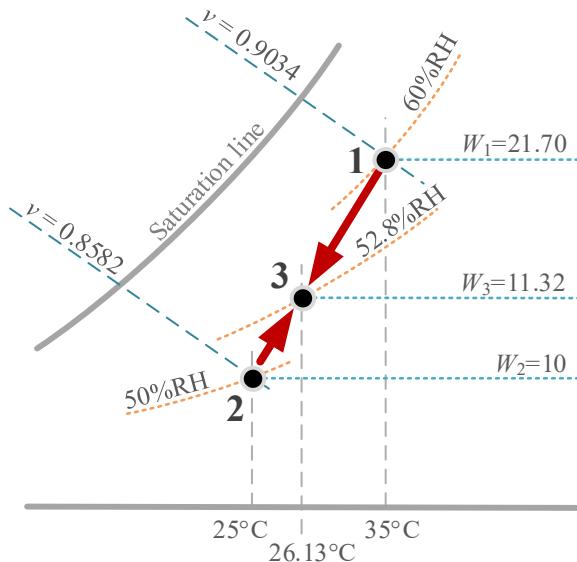
$$\dot{m}_{da3} = \dot{m}_{da1} + \dot{m}_{da2} = 0.2214 + 1.7478 = 1.9692 \text{ kg/s}$$

- The humidity ratio ( $W_3$ ) is calculated using Equation (55).

$$W_3 = \frac{\dot{m}_{da1}W_1 + \dot{m}_{da2}W_2}{\dot{m}_{da3}}$$

$$= \frac{0.2214 \times 21.7 + 1.7478 \times 10.0}{1.9692}$$

$$= 11.32 \text{ g/kg}_{da}$$



**Fig 1.16(SI)** Schematic Solution for Example 1.5

- The dry-bulb temperature ( $t_3$ ) is calculated using Equation (57).

$$t_3 = \frac{\dot{m}_{da1}t_1 + \dot{m}_{da2}t_2}{\dot{m}_{da3}} = \frac{0.2214 \times 35 + 1.7478 \times 25}{1.9692} = 26.13^\circ\text{C}$$

From the result,  $t_3 = 26.13^\circ\text{C}$  and  $W_3 = 11.32 \text{ g/kg}_{da}$ , the relative humidity at State 3 is 52.8%RH. According to the psychrometric chart, a line drawn from State 1 to State 2, intersected by a horizontal line at  $W_3$ , yields  $t_3$  and relative humidity values at State 3 that closely match the calculated results.

**Example 1.5****IP Units**

Outdoor air at 95°F dry-bulb and 60% RH at an airflow rate of 400 cfm is mixed with indoor air at 77°F dry-bulb and 50% RH at an airflow rate of 3000 cfm in the adiabatic mixing chamber of the air-handling unit (AHU). Determine the dry-bulb temperature and relative humidity of the mixed air.

Given: 95°F, 60%RH,  $Q_1 = 400$  cfm,  $t_2 = 77^\circ\text{F}$ , 50%RH,  $Q_2 = 3000$  cfm,

$$p = 14.696 \text{ psia}$$

Required:  $t_2$  and  $\phi_2$

Solution: From spreadsheet calculations:  $14.47 \text{ ft}^3/\text{lb}_{da}$ ,  $W_1 = 0.0217 \text{ lb/lb}_{da}$ , and  $v_2 = 13.74 \text{ ft}^3/\text{lb}_{da}$ ,  $W_2 = 0.0100 \text{ lb/lb}_{da}$ .

- The dry air mass flow rate ( $\dot{m}_{da}$ ) is calculated by substituting  $v$  into Equation (42).

$$\dot{m}_{da1} = 60Q_1/v_1 = 60 \times 400/14.47 = 1659 \text{ lb/hr}$$

$$\dot{m}_{da2} = 60Q_2/v_2 = 60 \times 3000/13.74 = 13100 \text{ lb/hr}$$

$$\dot{m}_{da3} = \dot{m}_{da1} + \dot{m}_{da2} = 1659 + 13100 = 14759 \text{ lb/hr}$$

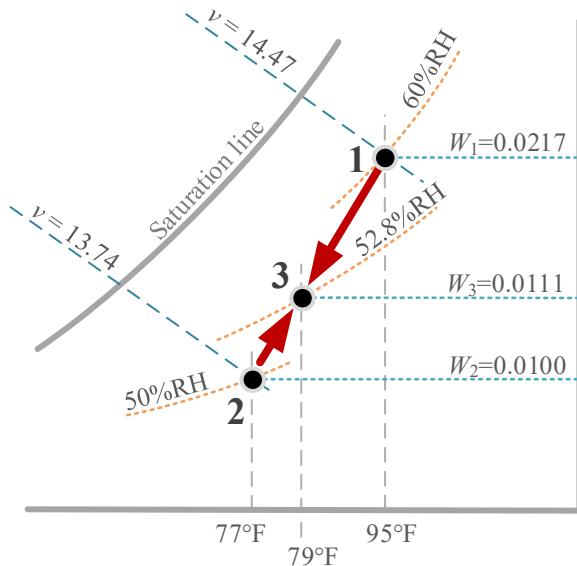
- The humidity ratio ( $W_3$ ) is calculated using Equation (55).

$$W_3 = \frac{\dot{m}_{da1}W_1 + \dot{m}_{da2}W_2}{\dot{m}_{da3}} = \frac{1659 \times 0.0217 + 13100 \times 0.01}{14759} = 0.01132 \text{ lb/lb}_{da}$$

- The dry-bulb temperature ( $t_3$ ) is calculated using Equation (57).

$$t_3 = \frac{\dot{m}_{da1}t_1 + \dot{m}_{da2}t_2}{\dot{m}_{da3}} = \frac{1659 \times 95 + 13100 \times 77}{14759} = 79.0^\circ\text{F}$$

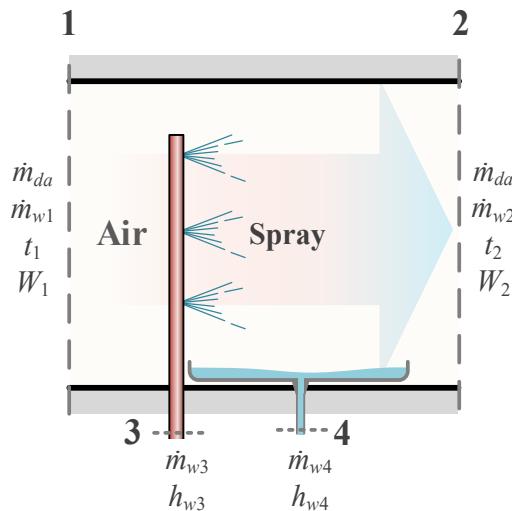
From the result,  $t_3 = 79^\circ\text{F}$  and  $W_3 = 0.01132 \text{ lb/lb}_{da}$ , the relative humidity at State 3 is 52.8% RH. According to the psychrometric chart, a line drawn from State 1 to State 2, intersected by a horizontal line at  $W_3$ , yields  $t_3$  and relative humidity values at State 3 that closely match the calculated results.



**Fig 1.16(IP)** Schematic Solution for Example 1.5

### 1.7.5 Adiabatic Mixing of Water Injected into Moist Air

Air-conditioning systems that require increased humidity use steam injection, which involves introducing steam into the air ducts of the air-handling unit. In the case of water spray, it is commonly used in air washers in textile factories, where low temperature and high humidity are required. This process is illustrated in [Figure 1.17](#).



**Fig. 1.17** Schematic for Adiabatic Mixing of Water Injected into Moist Air

*Mass balance equations:*

$$\text{dry air: } \dot{m}_{da1} = \dot{m}_{da2} = \dot{m}_{da} \quad \text{SI and IP} \quad (59)$$

$$\text{water vapor: } \dot{m}_{w3} + \dot{m}_{da}W_1 = \dot{m}_{w4} + \dot{m}_{da}W_2$$

$$\dot{m}_{w2} - \dot{m}_{w1} = \dot{m}_{w3} - \dot{m}_{w4} = \dot{m}_{da}(W_2 - W_1) \quad \text{SI and IP} \quad (60)$$

*First law of thermodynamics:*

Based on moist air property analysis and assuming no heat loss, the following relationship is obtained:

$$\dot{m}_{da1}h_1 + \dot{m}_{w3}h_{w3} = \dot{m}_{da2}h_2 + \dot{m}_{w4}h_{w4}$$

$$\dot{m}_{da}(h_2 - h_1) = \dot{m}_{w3}h_{w3} - [\dot{m}_{w3} - \dot{m}_{da}(W_2 - W_1)]h_{w4}$$

$$\dot{m}_{da}(h_2 - h_1) = \dot{m}_{w3}(h_{w3} - h_{w4}) + \dot{m}_{da}(W_2 - W_1)h_{w4} \quad \text{SI and IP} \quad (61)$$

**Water Spray Process:** In the water spray process, air may be heated, cooled, humidified, or dehumidified depending on the mean surface temperature of the water droplets ( $t_s$ ). The droplet surface is treated as a wet surface capable of transferring both sensible and latent heat. The direction of the process depends on temperature and vapor pressure. When analyzed on the psychrometric chart, as illustrated in [Figure 1.18](#), the following processes may occur:

**Process 1–2a: Heating and Humidification ( $t_s > t_1$ )**

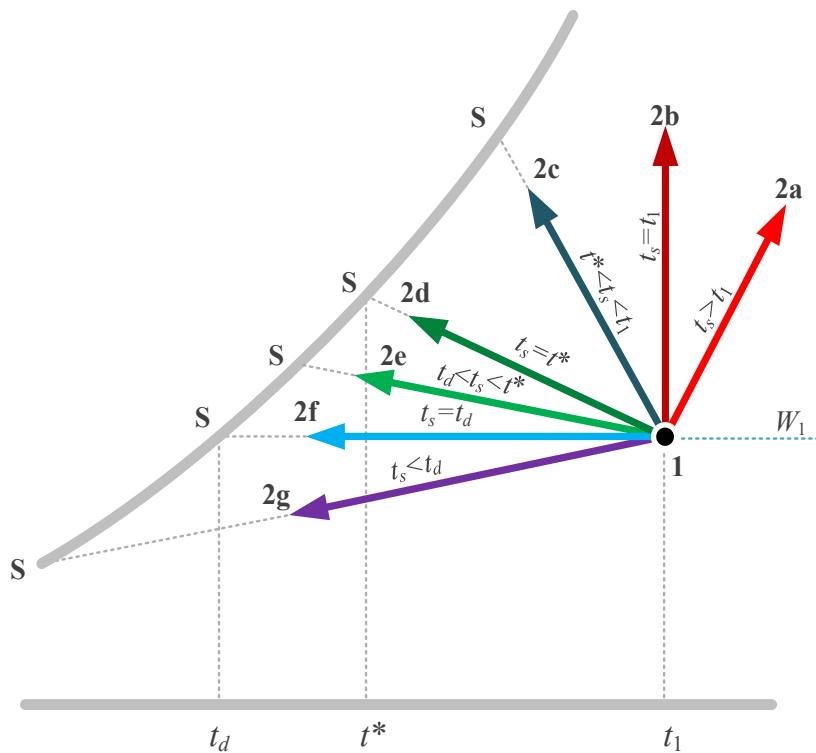
When  $t_s$  is greater than the air temperature at State 1, water transfers sensible heat to the air and simultaneously evaporates, adding moisture. As a result, the air becomes warmer and more humid.

**Process 1–2b: Humidification Only ( $t_s = t_1$ )**

When  $t_s$  equals the air temperature at State 1, no sensible heat is transferred. However, evaporation still occurs, adding moisture and increasing air humidity.

**Process 1–2c: Cooling and Humidification ( $t^* < t_s < t_1$ )**

When  $t^* < t_s < t_1$ , water transfers sensible heat (sensible cooling) and evaporates into the air. As a result, the air is cooled and humidified. The final air state has higher enthalpy.



**Fig 1.18** Water Spray Process

#### Process 1–2d: Cooling and Humidification ( $t_s = t^*$ )

When  $t_s$  equals the wet-bulb temperature ( $t^*$ ) at State 1, the air is cooled and humidified, as in the previous case. The process follows a line of constant wet-bulb temperature on the psychrometric chart.

#### Process 1–2e: Cooling and Humidification ( $t_d < t_s < t^*$ )

When  $t_d < t_s < t^*$ , the process is similar to Process 1–2c. However, the final air state has lower enthalpy.

#### Process 1–2f: Cooling ( $t_s = t_d$ )

When  $t_s$  equals the dew point temperature ( $t_d$ ) of the air at State 1, the water transfers sensible heat to the air but does not evaporate into it.

#### Process 1–2g: Cooling and Dehumidification ( $t_s < t_d$ )

When  $t_s$  is lower than the dew point temperature ( $t_d$ ) at State 1, water transfers sensible heat to the air, causing water vapor in the air to condense into droplets. As a result, the air is cooled and dehumidified, similar to a typical cooling process.

**Evaporative cooling or steam humidification processes:** In cases where steam or fine water mist is introduced using nozzles that allow complete evaporation, the outgoing water mass term is negligible. Equations (60) and (61) are simplified as follows:

$$\dot{m}_{da}(h_2 - h_1) = \dot{m}_w h_w = \dot{m}_{da}(W_2 - W_1)h_{w3}$$

From the equations, if initial air conditions and the mass flow rate of water ( $\dot{m}_w$ ) are known,  $W_2$  and  $h_2$  can be determined. These values can be used to locate the intersection point on the psychrometric chart, from which outlet air properties are determined. However, in many practical cases, only the final temperature or humidity may be known, while  $\dot{m}_w$  is unknown. In such cases, solving the equations directly becomes impractical, and a semicircular protractor on the psychrometric chart is required.

From Equations (60) and (61):

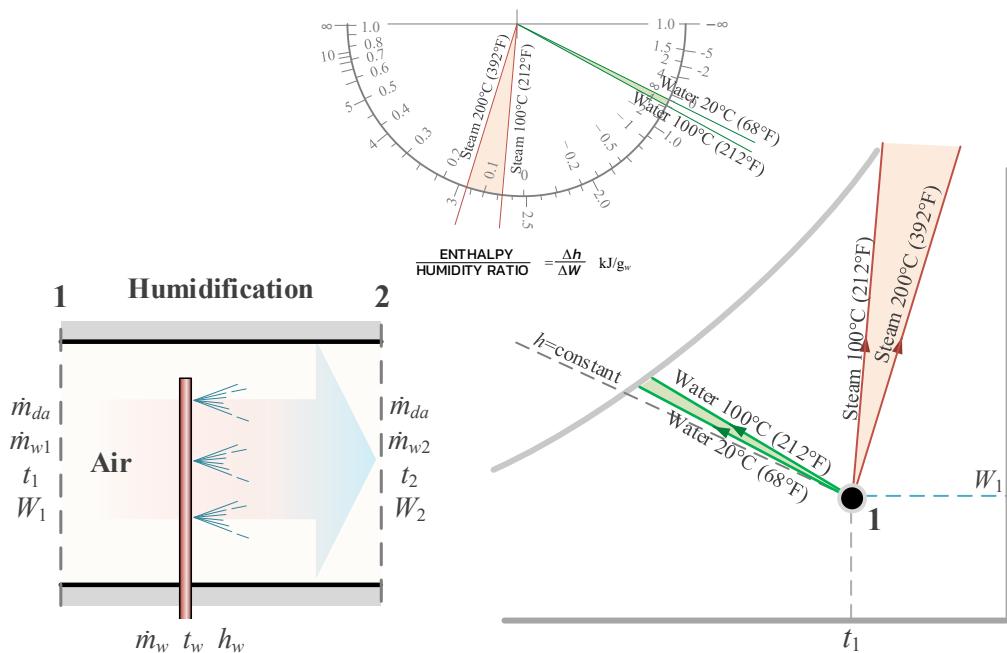
$$\frac{h_2 - h_1}{W_2 - W_1} = h_w$$

Alternatively, it is expressed as  $\Delta h/\Delta W$ , as follows:

$$\frac{\Delta h}{\Delta W} = h_w \quad \text{SI and IP} \quad (62)$$

According to the equation, the value of  $h_w$  is equal to  $\Delta h/\Delta W$ , as indicated on the semi-circular protractor of the psychrometric chart. Therefore, if we know the thermodynamic properties of the water or steam we are introducing, we draw a line on the protractor as a process reference. In the case where water is sprayed, the process follows the 1–2d path shown in [Figure 1.18](#). However, if steam with a high enthalpy ( $h_g$ ) is used, the reference line slopes toward the 1–2a path, as illustrated in [Figure 1.19](#).

According to Equation (62),  $\Delta W$  is very tiny, and when multiplied by  $h_w$ , the result is also negligible. Therefore,  $\Delta h$  is small, implying that  $h_2$  and  $h_1$  are nearly equal, or  $h \approx \text{constant}$ . This behavior resembles a wet-bulb temperature process. Therefore, the wet-bulb temperature during water spray processes tends to remain nearly constant. For water misting fans that spray water into the air, the process reduces air temperature and increases humidity. However, it is not an adiabatic process because heat transfer occurs across the control volume.



**Fig. 1.19** Process Lines for Adiabatic Water Spray or Steam Injection

When estimating  $t_2$  is required, the moist air enthalpy equation (Equation 32) can be substituted into Equation (61), resulting in

$$1.006t_2 + W_2(2501 + 1.86t_2) - (1.006t_1 + W_1(2501 + 1.86t_1)) = \Delta Wh_w \quad \text{SI}$$

$$1.006(t_2 - t_1) + 2501\Delta W + 1.86(t_2 W_2 - t_1 W_1) = \Delta Wh_w \quad \text{SI}$$

$$0.240t_2 + W_2(1061 + 0.444t_2) - (0.240t_1 + W_1(1061 + 0.444t_1)) = \Delta Wh_w \quad \text{IP}$$

$$0.240(t_2 - t_1) + 1061\Delta W + 0.444(t_2 W_2 - t_1 W_1) = \Delta Wh_w \quad \text{IP}$$

where  $\Delta W = W_2 - W_1$

To estimate  $t_2$ ,  $1.86(t_2 W_2 - t_1 W_1) \approx 1.86t_m\Delta W$  in SI units, or  $0.444(t_2 W_2 - t_1 W_1) \approx 0.444t_m\Delta W$  in IP units, is assumed. For typical air-conditioning processes,  $t_m \approx 24^\circ\text{C}$  ( $75^\circ\text{F}$ ) is used. Substituting these into the equation yields the following expression:

$$t_2 \approx t_1 + (h_w - 2546)\Delta W \quad \text{SI} \quad (63)$$

$$t_2 \approx t_1 + (h_w - 1094)\Delta W/0.24 \quad \text{IP} \quad (63)$$

The result from this equation is a preliminary estimate only. Note that the result is invalid if State 2 lies beyond the saturation line.

**Example 1.6****SI Units**

Outdoor air at  $44^{\circ}\text{C}$  dry-bulb, 24% RH with an airflow rate of 3600  $\text{m}^3/\text{hr}$  flows through a water misting fan. To reduce the air temperature to  $28^{\circ}\text{C}$ , determine the amount of water required.

Given:  $t_1 = 44^{\circ}\text{C}$ , 24%RH,  $Q_1 = 3600/3600 = 1.0 \text{ m}^3/\text{s}$ ,  $t_2 = 28^{\circ}\text{C}$ .

Required:  $\dot{m}_w$ .

Assumption:  $p = 101.325 \text{ kPa}$ , sprayed water temperature ( $t_w$ ) =  $27^{\circ}\text{C}$ ; assume an adiabatic process.

**Solution:** From spreadsheet calculations, we yield the following values:

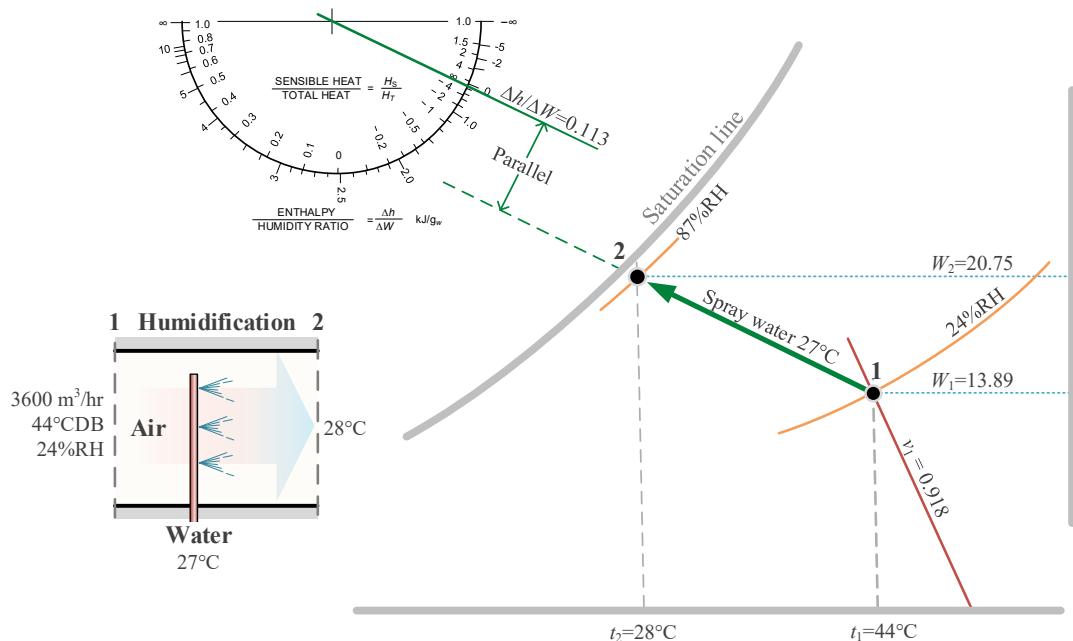
$$v_1 = 0.918 \text{ m}^3/\text{kg}_{da}, W_1 = 13.89 \text{ g}_w/\text{kg}_{da}, h_1 = 80.13 \text{ kJ/kg}_{da},$$

- The inlet enthalpy ( $h_w$ ) and mass flow rate ( $\dot{m}_{da}$ ) are determined as follows:

$$h_w = 4.186t_w = 4.186 \times 27 = 113.0 \text{ kJ/kg}_w \text{ or } 0.113 \text{ kJ/g}_w$$

$$\dot{m}_{da} = Q_1/v_1 = 1/0.918 = 1.089 \text{ kg/s}$$

- Equation (62) yields  $\Delta h/\Delta W = h_w = 0.113 \text{ kJ/g}_w$ . This value is used to draw a reference line on the semicircular protractor. The process line is then drawn from State 1, parallel to this reference line, intersecting the  $t_2 = 28^{\circ}\text{C}$  dry-bulb on the psychrometric chart. The resulting process is illustrated in [Figure 1.20](#).



**Fig. 1.20(SI)** Schematic Solution for Example 1.6

- The psychrometric chart indicates that the humidity ratio at State 2  $W_2 = 0.02075$   $\text{lb}_w/\text{lb}_{da}$  at 87% RH. Equation (60) then uses this value to calculate the amount of water spray.

$$\dot{m}_w = \dot{m}_{da}(W_2 - W_1) = 1.089(0.02075 - 0.01324)$$

$$\dot{m}_w = 7.47 \text{ g/s}$$

**Example 1.6**
**IP Units**

Outdoor air at 110°F dry-bulb, 24% RH with an airflow rate of 2000 cfm flows through a water misting fan. To reduce the air temperature to 82°F, determine the amount of water required.

Given: 110°F, 24% RH,  $Q_1 = 2000$  cfm,  $t_2 = 82^\circ\text{F}$ ,  $p = 14.696$  psia.

Required:  $\dot{m}_w$

Assumption:  $p = 14.696$  psia, sprayed water temperature ( $t_w$ ) = 80°F; assume an adiabatic process.

Solution: From spreadsheet calculations, we yield the following values:

$$v_1 = 14.67 \text{ ft}^3/\text{lb}_{da}, W_1 = 0.01324 \text{ lb}_w/\text{lb}_{da}, h_1 = 41.1 \text{ Btu/lb}_{da}.$$

- The inlet enthalpy ( $h_w$ ) and mass flow rate ( $\dot{m}_{da}$ ) are determined as follows:

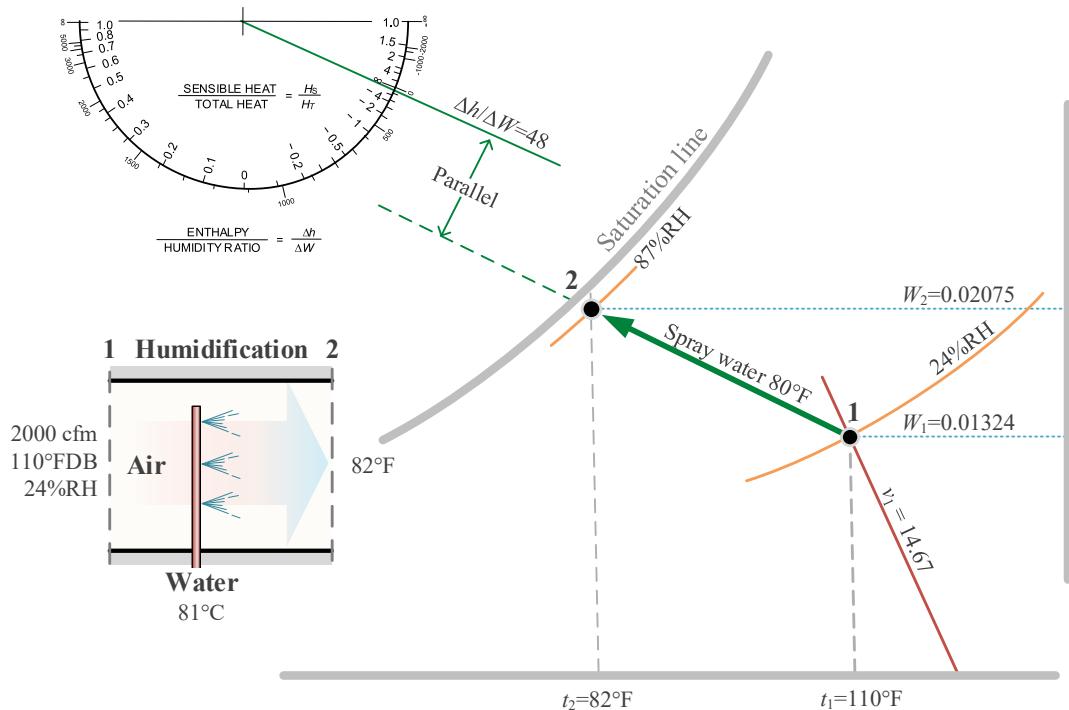
$$h_w = t_w - 32 = 80 - 32 = 48 \text{ Btu/lb}_w$$

$$\dot{m}_{da} = 60Q_1/v_1 = 60 \times 2000/14.67 = 8180 \text{ lb}_{da}/\text{hr}$$

- Equation (62) yields  $\Delta h/\Delta W = h_w = 48 \text{ Btu/lb}_{da}$ . This value is used to draw a reference line on the semi-circular protractor. The process line is then drawn from State 1, parallel to this reference line, intersecting  $t_2 = 82^\circ\text{F}$  dry-bulb on the psychrometric chart. The resulting process is illustrated in [Figure 1.20](#).
- The psychrometric chart indicates that the humidity ratio at State 2 is  $W_2 = 0.02075 \text{ lb}_w/\text{lb}_{da}$ , at 87% RH. Equation (60) then uses this value to calculate the amount of water spray.

$$\dot{m}_w = \dot{m}_{da}(W_2 - W_1) = 8180(0.02075 - 0.01324)$$

$$\dot{m}_w = 61.43 \text{ lb}_w/\text{hr}$$



**Fig. 1.20(IP)** Schematic Solution for Example 1.6

**Example 1.7**

**SI Units**

Air at 24°C dry-bulb, 30%RH with an airflow rate of 3600 m<sup>3</sup>/hr is to be humidified to 50%RH by injecting steam at 110°C into the system. Determine the required steam mass flow rate and final air temperature.

Given:  $t_1 = 24^\circ\text{C}$ ,  $\phi_1 = 30\% \text{ RH}$ ,  $Q_1 = 3600/3600 = 1.0 \text{ m}^3/\text{s}$ ,  $\phi_2 = 50\% \text{ RH}$ ,  $t_w = 110^\circ\text{C}$

Required:  $\dot{m}_w$  and  $t_2$ .

Assumption:  $p = 101.325 \text{ kPa}$ .

Solution: From spreadsheet calculations, we yield the following values:

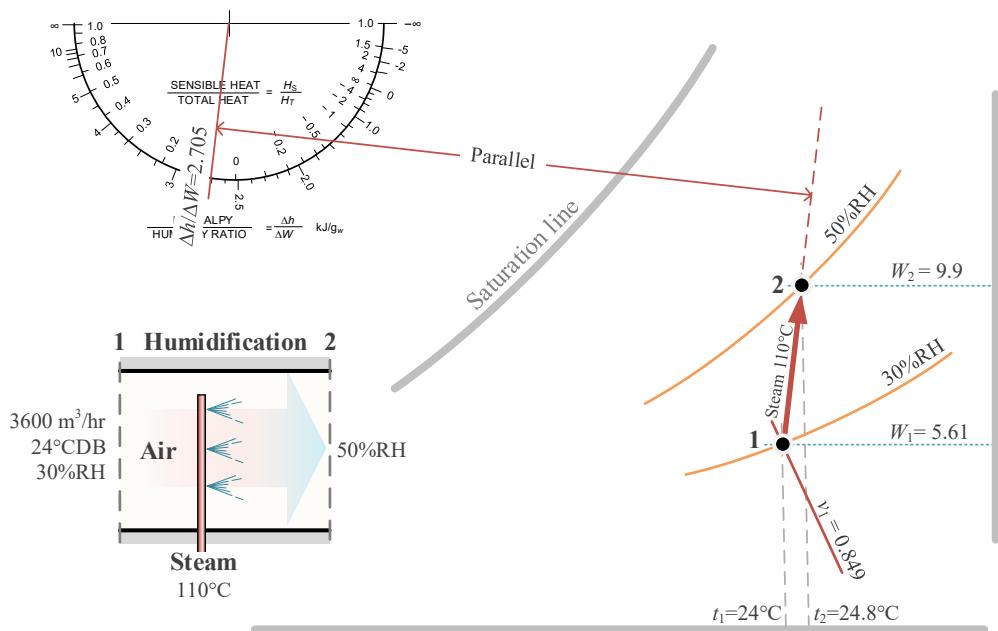
$$v_1 = 0.849 \text{ m}^3/\text{kg}_{da}, W_1 = 5.61 \text{ g}_w/\text{kg}_{da}, h_1 = 38.42 \text{ kJ/kg}_{da}$$

- From enthalpy equations:

$$\begin{aligned}
 h_w &= h_g = 2501 + 1.86t_w \\
 &= 2501 + 1.86 \times 110 \\
 &= 2705 \text{ kJ/kg}_w \text{ or } 2.705 \text{ kJ/g}_w
 \end{aligned}$$

and  $\dot{m}_{da} = Q_1/v_1 = 1/0.849 = 1.177 \text{ kg/s}$

- Equation (62) yields  $\Delta h/\Delta W = h_g = 2.705 \text{ kJ/g}_w$ . This value is used to draw a reference line on the semicircular protractor. The process line is then drawn from State 1, parallel to this reference line, intersecting the 50% RH line on the psychrometric chart. The result is shown in [Figure 1.21](#).



**Fig. 1.21(SI)** Schematic Solution for Example 1.7

- From the psychrometric chart,  $t_2 = 24.8^\circ\text{C}$  and  $W_2 = 9.9 \text{ g}_w/\text{kg}_{da}$ . Substituting these into Equation (60), the required steam mass flow rate is determined as follows:

$$\dot{m}_w = \dot{m}_{da}(W_2 - W_1) = 1.177(9.9 - 5.61)$$

$$\dot{m}_w = 5.051 \text{ g/s}$$

**Example 1.7****IP Units**

Air at 75°F dry-bulb, 30%RH with an airflow rate of 2000 cfm is to be humidified to 50%RH by injecting steam at 230°F into the system. Determine the required steam mass flow rate and final air temperature.

Given:  $t_1 = 75^\circ\text{F}$ ,  $\phi_1 = 30\%$  RH,  $Q_1 = 2000$  cfm,  $\phi_2 = 50\%$  RH,  $t_w = 230^\circ\text{F}$ .

Required:  $\dot{m}_w$  and  $t_2$ .

Assumption:  $p = 14.696$  psia.

Solution: From spreadsheet calculations, we yield the following values:

$$v_1 = 13.6 \text{ ft}^3/\text{lb}_{da}, W_1 = 0.00551 \text{ lb}_w/\text{lb}_{da}, h_1 = 24.03 \text{ Btu/lb}_{da}.$$

- From the enthalpy equations:

$$\begin{aligned} h_w &= h_g = 1060 + 0.44t_w \\ &= 1060 + 0.44 \times 230 \\ &= 1161.2 \text{ Btu/lb}_w \end{aligned}$$

$$\text{and} \quad \dot{m}_{da} = 60Q_1/v_1 = 60 \times 2000/13.6 = 8824 \text{ lb}_{da}/\text{hr}$$

- Equation (62) yields  $\Delta h/\Delta W = h_g = 1161.2 \text{ Btu/lb}_w$ . This value is used to draw a reference line on the semi-circular protractor. The process line is then drawn from State 1, parallel to this reference line, intersecting the 50% RH line on the psychrometric chart. The result is shown in [Figure 1.21](#).
- From the psychrometric chart,  $t_2 = 76.6^\circ\text{F}$  and  $W_2 = 0.0099 \text{ lb}_w/\text{lb}_{da}$ . Substituting these into Equation (60), The required steam mass flow rate is determined as follows:

$$\dot{m}_w = \dot{m}_{da}(W_2 - W_1) = 8824(0.0099 - 0.00551)$$

$$\dot{m}_w = 38.74 \text{ lb}_w/\text{hr}$$

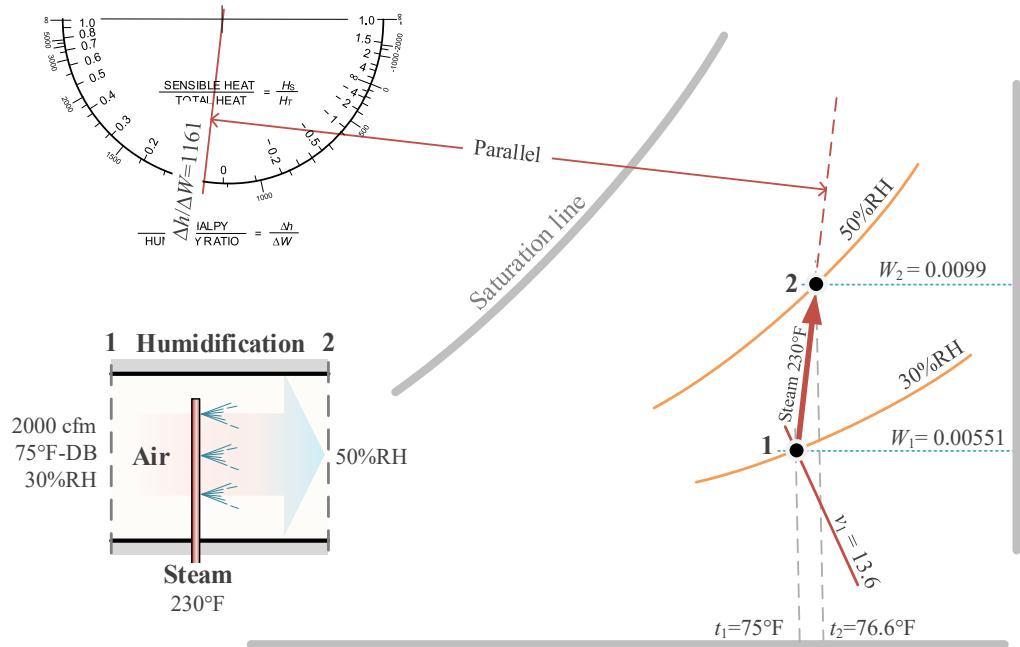


Fig. 1.21(IP) Schematic Solution for Example 1.7

### 1.7.6 Adiabatic Heating and Humidification

Heating and humidification processes occur commonly in high-precision air-conditioning systems, where both heat and moisture may require compensation under part-load conditions. These processes can be analyzed separately—one for heating and one for steam injection—or combined into a single analysis. In the case of combined analysis, the process is described by the following equation:

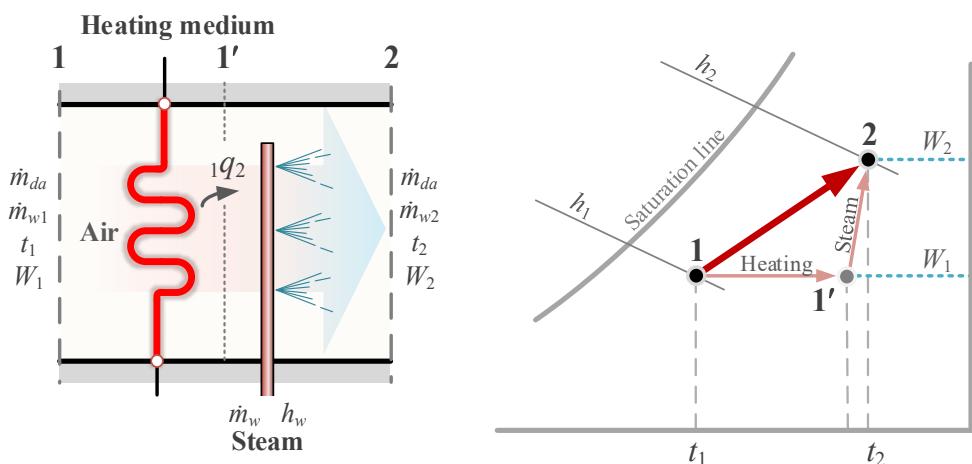


Fig. 1.22 Schematic for Adiabatic Heating and Humidification Process

*Mass balance equations:*

dry air:  $\dot{m}_{da1} = \dot{m}_{da2} = \dot{m}_{da}$  SI and IP (64)

water vapor:  $\dot{m}_{da1}W_1 + \dot{m}_w = \dot{m}_{da2}W_2$  SI and IP (65)

*First law of thermodynamics:*

$$\dot{m}_{da1}h_1 + \dot{m}_w h_w + 1q_2 = \dot{m}_{da2}h_2 \quad \text{SI and IP} \quad (66)$$

By combining Equations (64), (65), and (66), the following expressions are obtained:

$$\frac{\Delta h}{\Delta W} = 1q_2/\dot{m}_w + h_w \quad \text{SI and IP} \quad (67)$$

Equation (67) is still expressed in the form of  $\Delta h/\Delta W$  and shown on the semi-circular protractor of the psychrometric chart.

**Example 1.8**

**SI Units**

Air at 13°C dry-bulb, 90%RH with an airflow rate of 7200 m<sup>3</sup>/h flows through a device that heats and humidifies the air by injecting steam at 110°C until the air reaches 20°C dry-bulb and 65% RH. Determine the amount of steam injected, heat added to the air, and air temperature exiting the heating coil.

Given:  $t_1 = 13^\circ\text{C}$ ,  $\phi_1 = 90\%$  RH,  $Q_1 = 7200/3600 = 2.0 \text{ m}^3/\text{s}$ ,  $t_2 = 20^\circ\text{C}$ ,  $\phi_2 = 65\%$  RH,  $t_w = 110^\circ\text{C}$ ,  $p = 101.325 \text{ kPa}$ .

Required:  $\dot{m}_w$ ,  $1q_2$  and  $t_1'$ .

Solution: From spreadsheet calculations, we yield the following values:

$$v_1 = 0.822 \text{ m}^3/\text{kg}_{da}, W_1 = 8.48 \text{ g}_w/\text{kg}_{da}, h_1 = 34.49 \text{ kJ/kg}_{da}, \\ W_2 = 9.58 \text{ g}_w/\text{kg}_{da}, h_2 = 44.44 \text{ kJ/kg}_{da}.$$

- From the enthalpy equations:

$$h_w = h_g = 2501 + 1.86t_w = 2501 + 1.86 \times 110 = 2705 \text{ kJ/kg}_w \text{ or } 2.705 \text{ kJ/g}_w$$

$$\text{and } \dot{m}_{da} = Q_1/v_1 = 2.0/0.822 = 2.43 \text{ kg/s}$$

- From Equation (65), the required flow rate of steam is determined as follows:

$$\dot{m}_w = \dot{m}_{da}(W_2 - W_1) = 2.43(9.58 - 8.48) = 2.68 \text{ g/s or } 0.00268 \text{ kg/s}$$

- Equation (66) determines the added heat to the air as follows:

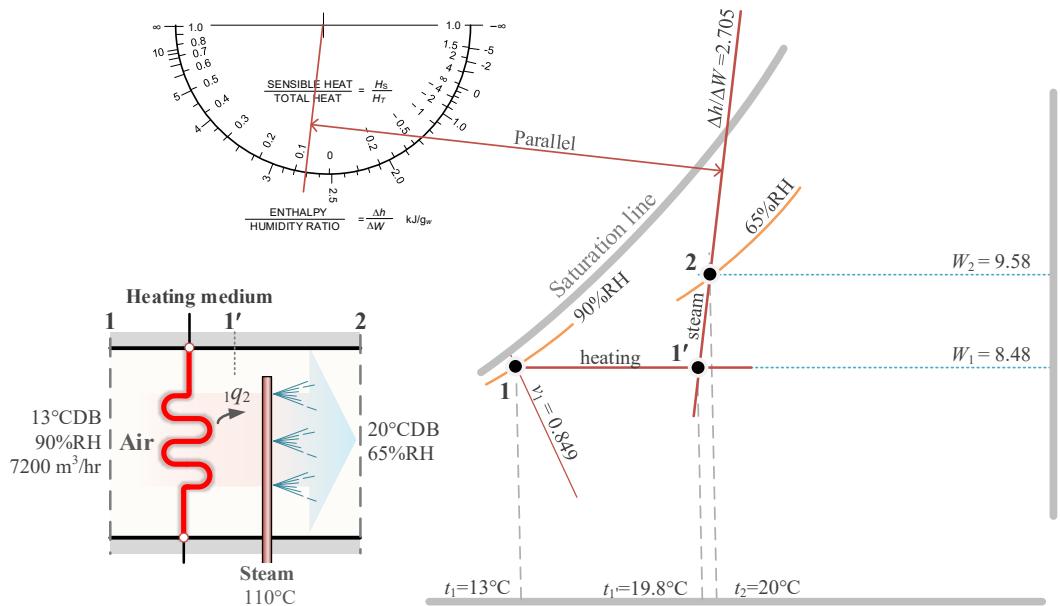
$$1q_2 = \dot{m}_{da2}h_2 - \dot{m}_{da1}h_1 - \dot{m}_w h_w = 2.43(44.44 - 34.49) - 0.00268 \times 2705 \\ 1q_2 = 16.96 \text{ kW}$$

- From the sensible heat equation (Equation 39):

$$1q_2 = q_s = \dot{m}_{da} (1.006 + 1.86W)(t_{1'} - t_1)$$

$$t_{1'} = 13 + 16.96/[2.43 \times (1.006 + 1.86 \times 8.48/1000)] = 19.82^\circ\text{C}$$

- A quick alternative to estimate  $t_{1'}$  uses the semi-circular protractor together with separating the heating and steam injection processes. The procedure is as follows:
  - 1) Draw a reference line based on  $\Delta h/\Delta W = h_g = 2.705 \text{ kJ/g}_w$ , then draw the steam injection process line from State 2, parallel to this reference line, as illustrated in [Figure 1.23](#).
  - 2) Draw the sensible heating process line from State 1 to intersect the steam injection process line. The point of intersection is denoted as State  $1'$ , as shown in [Figure 1.23](#).
  - 3) Read  $t_{1'}$  to calculate  $1q_2$  using Equation (39).



**Fig. 1.23(SI)** Schematic Solution for Example 1.8

**Example 1.8****IP Units**

Air enters at 55°F dry-bulb, 90%RH with an airflow rate of 4000 cfm and flows through a device that heats and humidifies the air by injecting steam at 230°F until the air reaches 68°F dry-bulb and 65% RH. Determine the amount of steam injected, heat added to the air, and air temperature exiting the heating coil.

Given:  $t_1 = 55^\circ\text{F}$ ,  $\phi_1 = 90\%$  RH,  $Q_1 = 4000 \text{ cfm}$ ,  $t_2 = 68^\circ\text{F}$ ,  $\phi_2 = 65\%$  RH,  $t_w = 230^\circ\text{F}$ ,  $p = 14.696 \text{ psia}$ .

Required:  $\dot{m}_w$ ,  $q_2$  and  $t_{1'}$ .

Solution: Spreadsheet calculations, we yield the following values:

$$v_1 = 13.08 \text{ ft}^3/\text{lb}_{da}, W_1 = 0.00848 \text{ lb}_w/\text{lb}_{da}, h_1 = 14.82 \text{ Btu/lb}_{da}, \\ W_2 = 0.00958 \text{ lb}_w/\text{lb}_{da}, h_2 = 19.10 \text{ Btu/lb}_{da}.$$

- From the enthalpy equations:

$$h_w = h_g = 1061 + 0.444t_w = 1061 + 0.444 \times 230 = 1163 \text{ Btu/lb}_w$$

$$\dot{m}_{da} = 60Q_1/v_1 = 60 \times 4000/13.08 = 18349 \text{ lb}_{da}/\text{hr}$$

- From Equation (65), the required flow rate of steam is determined as follows:

$$\dot{m}_w = \dot{m}_{da}(W_2 - W_1) = 18349(0.00958 - 0.00848) = 20.18 \text{ lb}_w/\text{hr}$$

- Equation (66) determines the added heat to the air as follows:

$$1q_2 = \dot{m}_{da}h_2 - \dot{m}_{da}h_1 - \dot{m}_w h_w = 18349(19.10 - 14.82) - 20.18 \times 1163$$

$$1q_2 = 55064 \text{ Btu/h}$$

- From the sensible heat equation (Equation 39):

$$1q_2 = q_s = \dot{m}_{da} (0.24 + 0.444w) (t_{1'} - t_1)$$

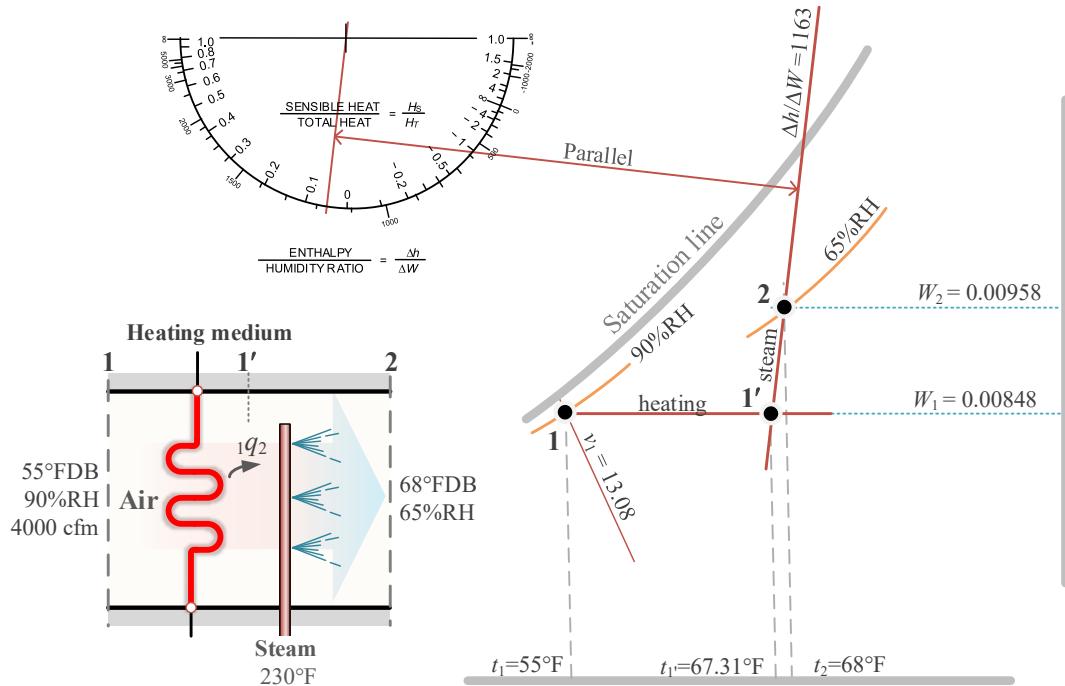
$$t_{1'} = 55 + 55064/[18349 \times (0.24 + 0.444 \times 0.00848)]$$

$$t_{1'} = 67.31^\circ\text{F}$$

- A quick alternative to estimate  $t_{1'}$  uses the semi-circular protractor together with separating the heating and steam injection processes. The procedure is as follows:

- 1) Draw a reference line based on  $\Delta h/\Delta W = h_g = 1163 \text{ Btu/lb}_w$ , then draw the steam injection process line from State 2, parallel to this reference line, as illustrated in [Figure 1.23](#).

- 2) Draw the sensible heating process line from State 1 to intersect the steam injection process line. The point of intersection is denoted as State 1', as shown in [Figure 1.23](#).
- 3) Read  $t_{1'}$  to calculate  $1q_2$  using Equation (39).



**Fig. 1.23(IP)** Schematic Solution for Example 1.8

This method suits users of the psychrometric chart as a tool for analysis. Once you have obtained  $t_{1'}$ . This estimate may deviate slightly due to limitations in chart reading.