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PYTHON

for Mathematics & Computations

Pramote Dechaumphai

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Preface

Python is a popular freeware known for its user-friendliness. The software contains efficient packages for solving mathematical and computational problems. These packages include mathematical symbol packages, which facilitate learning calculus and differential equations. The numerical computing packages offer various commands, making it easier to learn numerical methods in science and engineering. Additionally, the plotting package for displaying results is easy to handle. With short commands, users can plot graphs in different formats, enabling a quicker understanding of the physical meaning of the problem. Overall, Python is an excellent tool for those seeking an efficient and easy-to-use software for solving a wide range of scientific and engineering problems.

This book contains essential commands to solve basic mathematical and computational problems. The commands, along with examples provided in the book, will help students understand how to use this Python software more efficiently. The author would like to thank Chulalongkorn University Press for supporting the publication of this book for the benefit of students and those who realize the importance of this Python software.

Pramote Dechaumphai

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Chapter 1

Fundamentals

1.1 Introduction

Python is a widely used software nowadays because it has high capability in mathematics and computation. It is an easy-to-use software and can be downloaded without any cost. The software is popular because it contains pre-built modules that can provide immediate results without the need to write subroutines, as was done in the past. These pre-built modules are useful for learning mathematics and computation. For example, there are modules for finding the general solution of differential equations, for integrating functions in symbolic or numerical form, and for solving systems of algebraic equations. These modules are essential for obtaining results in current mathematical and engineering study. Students who use this software can easily solve difficult problems or complete homework assignments in various subjects.

1.2 Python Software

Python software was created by Guido Van Rossum at a mathematics and computing research institute in the Netherlands in the 1980s. Since then, it has become increasingly well-known and

widely used, developing into the Python 2 and Python 3 versions in 2000 and 2008, respectively. Currently, Python 3 is now used in academic, research, and development fields, as well as in various industrial applications.

Python software is a freeware that can be downloaded from the website https://www.python.org. The installation process is quite simple by following the steps recommended on the website.

After successfully installing Python software on a computer, there are two modes in which it can be used: (1) interactive mode and (2) script mode. In the interactive mode, users can directly enter commands, while in the script mode, they can combine commands into a script file for Python to execute all at once. This is similar to using subroutines or function routines in other programming languages.

Nowadays there are various Python shells available in different formats, making it convenient to use the software on Windows. Python shells include IDLE (Integrated Development Environment), Notepad, PyCharm, Anaconda, and more.

If users have previously used MATLAB software, they will find that Python software and MATLAB are very similar, whether it is the programming language or the use of prepackaged algorithms and plotting. MATLAB has these prepackaged algorithms built into the software, and users simply call them to use. Python, on the other hand, does not have these prepackaged algorithms and must call similar packages from outside. The important pre-packages for mathematics and computation include: (1) `numpy`, which is a numerical algorithm, (2) `math`, which is a standard mathematical function, (3) `sympy`, which is a package related to symbolic mathematics, and (4) `matplotlib`, which is a package for plotting graphs in two and three dimensions, and so on. Users must download and install these packages on their computer before using them. The process of downloading and installing these packages can be done conveniently by following the instructions on their respective websites.

In order for readers to understand on how to use Python to solve mathematical and computational problems effectively, this book presents key commands input through the keyboard to obtain

results quickly on the computer screen. Users can gain more experience on using the software after becoming familiar with the basic commands.

1.3 Basic Arithmetic Commands

One basic ability of Python is to easily perform mathematical calculations that are commonly used. This is similar to using a calculator to help calculate numbers. The basic mathematical operations that can be performed include addition, subtraction, multiplication, division, and exponentiation. For example, if we want to add or subtract two numbers, we can type the command such as,

```
>>> 2.813 + 5.892
8.705
>>> 9.657 - 4.425
5.232
```

For multiplication and division, we use the * and / symbols, respectively, in the usual order. As examples,

```
>>> 4.823 * 3.219
15.525237
>>> 14.893 / 5.207
2.8601882
```

For exponentiation, we use the ** symbol, such as,

```
>>> 8.2 ** 2
67.24
```

We can assign constant values to variables that we created, and then use the value of the variable in subsequent calculations. As an example,

```
>>> a = 5
>>> a + 4
9
```

We can also assign multiple variable values at once in a single line of code by using the semi-colon `;` to separate the variable assignments. For example,

```
>>> a = 2; b = 3; c = 4
>>> a + b - c
1
```

Assigning values to variables should be done with care, as variables with different capitalization are considered different variables. For examples,

```
>>> d = 5; D = 2
>>> d + D
7
>>> d - D
3
```

Steps for mathematical calculation in Python follow the general standard of computation. The software prioritizes exponentiation, followed by multiplication and division, both of which are of equal importance. Then, addition and subtraction are performed. To illustrate this sequence of operations more clearly, we consider the following example,

```
>>> 10 - 3 ** 2 * 2 / 3 + 7
11.0
```

In practice, parentheses are often used to clarify the order of operations and make it more explicit. The program will calculate the values within the parentheses first, and then proceed with the order of operations outlined above. For example, if we want to calculate,

$$5^3\left(\frac{3}{5}+\frac{9}{2^3}\right)$$

we enter the command,

```
>>> (5**3)*((3/5) + (9/(2**3)))
215.625
```

1.4 Other Useful Commands

In this topic, we will learn mathematical functions commonly used in science and engineering computation. Examples of such functions include the trigonometric, logarithmic, and square root functions.

1.4.1 Trigonometric Functions

Python does not include commonly used mathematical constants and trigonometric functions by default. Therefore, we need to import packages that contain these constants and functions before we can use them. For example, if we want to use the value of pi, we have to import the math package first as follows.

```
>>> import math
>>> math.pi
3.141592653589793
```

If we want to find the sine value of 60 degrees, we enter,

```
>>> math.sin(math.pi/3)
0.8660254037844386
```

As shown above, to find the sine value of 60 degrees, we need to use the "`math.`" prefix before the constant or function. Additionally, the angle used with the trigonometric functions must be in radians. Similarly, we can use other trigonometric functions, such as finding the cosine value of 45 degrees.

```
>>> math.cos(math.pi/4)
0.7071067811865476
```

The inverse trigonometric functions, such as arcsine, arccosine and arctangent can be determined in the same way,

```
>>> math.asin(0.5)
0.5235987755982989
>>> math.acos(-0.3)
1.8754889808102941
>>> math.atan(1.5)
0.982793723247329
```

The hyperbolic functions, such as hyperbolic sine, hyperbolic cosine and hyperbolic tangent can be determined in the same way. For examples,

```
>>> math.sinh(3)
10.017874927409903
>>> math.cosh(2)
3.7621956910836314
>>> math.tanh(4)
0.999329299739067
```

1.4.2 Exponential and Logarithmic Functions

The exponential function is a function in the form of e^x. For example, if we want to find the value of e^3, we enter the command,

```
>>> math.exp(3)
20.085536923187668
```

As there are several bases for the logarithmic function, if we want to use the natural logarithm, we enter,

```
>>> math.log(1)
0.0
```

But if we want to use the base-10 logarithm function, we would enter,

```
>>> math.log10(10)
1.0
```

1.4.3 Finding Real Roots

The command for finding the square root is similar to those used in other software. For example, to find the square root of 7, we enter,

```
>>> math.sqrt(7)
2.6457513110645907
```

As for other roots, such as the cube root, we may use the exponentiation. As an example, if we want to find the cube root of 8, we enter,

```
>>> 8**(1/3)
2.0
```

1.4.4 More Useful Commands

To check which constants and functions are included in the `math` package, we enter the following commands,

```
>>> import math
>>> dir(math)
['__doc__', '__loader__', '__name__', '__package__',
'__spec__', 'acos', 'acosh', 'asin', 'asinh', 'atan',
'atan2', 'atanh', 'ceil', 'copysign', 'cos', 'cosh',
'degrees', 'e', 'erf', 'erfc', 'exp', 'expm1', 'fabs',
'factorial', 'floor', 'fmod', 'frexp', 'fsum', 'gamma',
'gcd', 'hypot', 'inf', 'isclose', 'isfinite', 'isinf',
'isnan', 'ldexp', 'lgamma', 'log', 'log10', 'log1p',
'log2', 'modf', 'nan', 'pi', 'pow', 'radians',
'remainder', 'sin', 'sinh', 'sqrt', 'tan', 'tanh',
'tau', 'trunc']
```

To get detailed information on how to use a specific function, such as the `sin(_)` function, we can use the `help(math.sin)` function. For example, to get help on how to use the `sin(_)` function, we type,

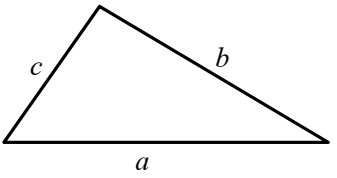
```
>>> help(math.sin)
Help on built-in function sin in module math:

sin(x, /)
    Return the sine of x (measured in radians).
```

1.5 Script File

The various Python commands that we type through the interactive mode as presented in the previous section can be collected and arranged into a script file. This allows for a one-time calculation through the script mode. The benefit of doing this is that

we do not have to type these commands repeatedly. The script file is similar to a subroutine or a function routine used in other languages to compute mathematical processes in the same format repeatedly. As a result, the length of the program is reduced, and it is also easier to debug the program.

The process of creating a script file can be easily done by opening the Python software as usual and selecting "New File" under the "File" menu at the top left corner of the screen. A new screen with an untitled file will appear. Then, we can start typing the commands we want. For example, the area of a triangle can be calculated from the lengths of all three sides as follows,



$$area = \sqrt{s(s-a)(s-b)(s-c)}$$

where $$s = \frac{a+b+c}{2}$$

If $a = 5$, $b = 4$, $c = 3$, then,

$$s = \frac{5+4+3}{2} = 6$$

Hence, the area of the triangle is,

$$area = \sqrt{6(6-5)(6-4)(6-3)} = 6$$

We can create a script file by typing it into this untitled file as follows,

```
import math
a = 5; b = 4; c = 3;
s = (a+b+c)/2
area = math.sqrt(s*(s-a)*(s-b)*(s-c))
print("%6.2f" %area)
```

After that, click "Run Module" under the "Run" menu, Python will prompt to save this file first. We can name it "`area`" and save it on the desktop of the computer. Then, click "Run Module", the result which is equal to `6.00` will appear in the main Python Shell window.

```
======= RESTART: C:/Users/Desktop/area.py =======
  6.00
```

Note that the Python script files always have a `.py` extension, which indicates that they contain Python code and can be executed by Python.

Creating a script file is quite simple. The software will alert if there are any errors in the script file. Debugging the program is also an easy task. We will learn more about programming in detail in the following sections.

1.6 Conclusion

In this chapter, we started by introducing the main components of Python software and basic usage, such as using it for simple mathematical calculations, including addition, subtraction, multiplication, division, and exponentiation. Then we introduced important mathematical functions commonly used in science or engineering, such as trigonometric functions, logarithmic functions, and exponential functions. We then learned some useful commands for working with Python software. Finally, we studied how to write simple programs using Python by simply collecting the commands needed in one file, which we can call and use them later. From the examples presented in this section, we see that Python is easy to use, not complicated, and can perform various tasks, as we will learn more in the following chapters.

Exercises

1. Use Python to determine the followings,

   (a) $4\frac{3}{5} + 5\frac{9}{8}$

   (b) $\frac{3}{5} + \frac{26}{7}$ (show answer in fraction form)

   (c) $43\left(\frac{\sqrt{513}}{6}+8^2\right) + \left(\frac{54}{7}+5.23^3\right)^{0.75}$

   (d) $\frac{\ln 150}{3.2} + 7.5^{\frac{2}{3}} - \left(\frac{\sqrt{135}+\sqrt[3]{250}}{e^{-0.3}}\right)^{\frac{1}{3}}$

   (e) $\sin^2\left(\frac{4\pi}{3}\right)\cos\left(\frac{5\pi}{6}\right) + \frac{\tan\left(\frac{\pi}{5}\ln 7\right)}{3+2\sqrt{3}}$

2. If $x = 8.67$, use Python to determine the followings,

   (a) $5x^3 + 2x^2 - 5x - 7.5$

   (b) $\frac{e^{\sqrt{x}} + \log_{10}(x+2)}{\sqrt[3]{0.02+3x^3}}$

   (c) $\frac{\sqrt{\sin x + \cos^2 x}}{3\log_{10}(x-2)}$ (show answer in fraction form)

   (d) $\frac{e^{1.58} - \sin 3x}{5} + \log_{10}(x+x^3)$

   (e) $\arctan(2x+3) - \cot(x\pi)$

3. Given $a = -5.73$, $b = 17.54$, $c = 2a + b/a$ and $d = 4ac + 3b$, use Python to determine the followings,

   (a) $c - \frac{d-a}{2c} + \frac{(a+c)^{2.5}}{\sqrt{abc}}$;

   (b) $\log\left|(a-c)(b-a)\right| + \frac{a-b+c-d}{a+b-c+d}$

(c) $a^2b - d^2c + \left(\frac{a}{c}\right)^2 - \left(\frac{b}{d}\right)^{0.5}$

4. Given the trigonometric relations,

(a) $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$

(b) $\cos 2x = 1 - 2\sin^2 x$

(c) $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

(d) $\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

(e) $\cot x = \frac{\text{cosec}^2 x}{1+\tan^2 x}$

Use Python to verify that the expressions above are valid by assuming appropriate numerical values of x.

5. A sphere has its radius of 21 cm, use Python to determine,

(a) size of the cube having the same surface area as the sphere

(b) size of the cube having the same volume as the sphere

6. Distance d from a point (x_0, y_0) to the line $Ax + By + C = 0$ is determined from,

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Use Python to determine the distance from the point (-4, 3) to the line $-2x + 7y - 10 = 0$.

7. Given,

$$\log_a N = \frac{\log_b N}{\log_b a}$$

(a) Employ `log(x)` to determine $\log_6 324$

(b) Employ `log10(x)` to determine $\log_3 1024$